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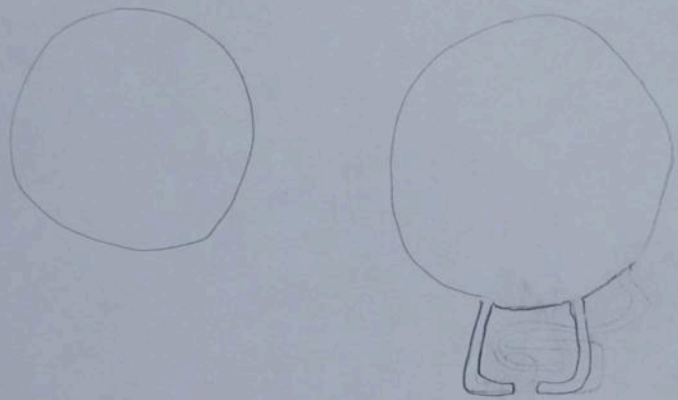
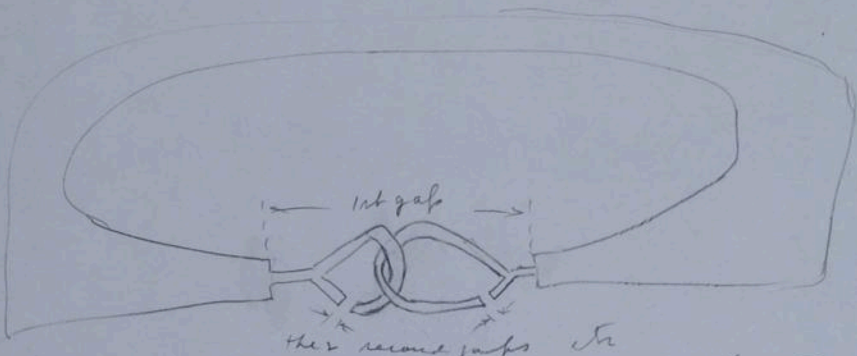
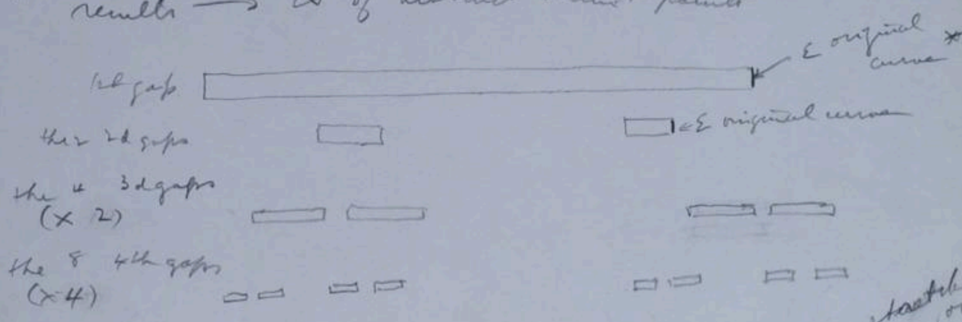


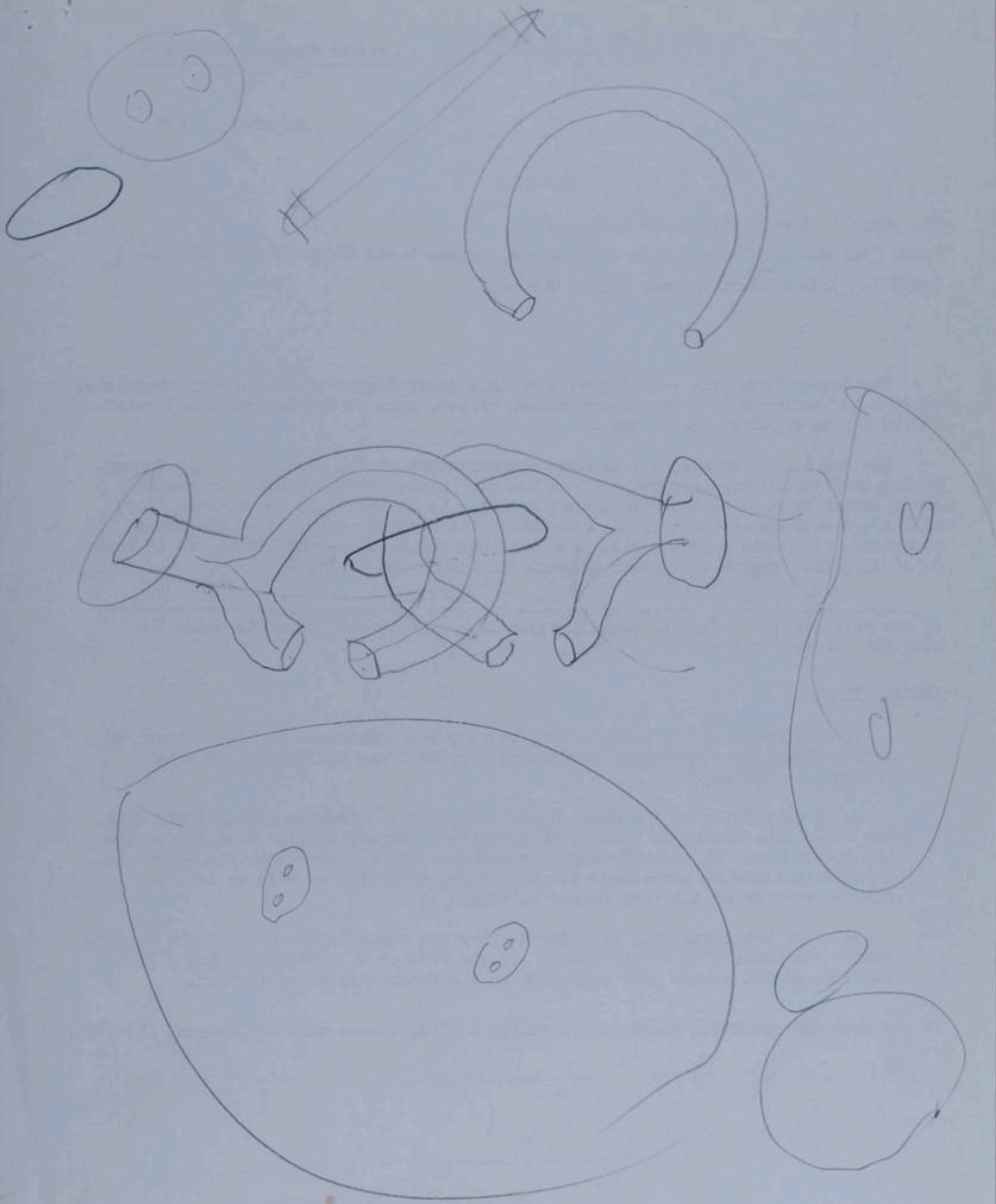


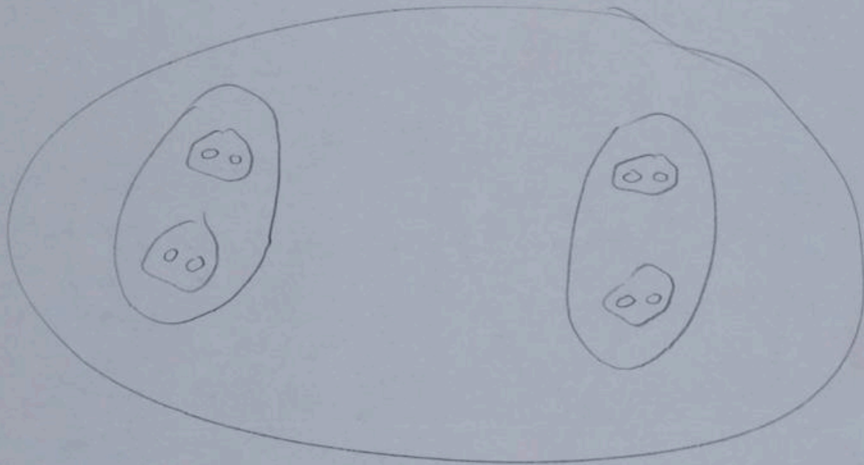
Suitably set up for say a multiple Einschachtelung of decreasing gaps* results ⟶ ∞ of distinct limit points

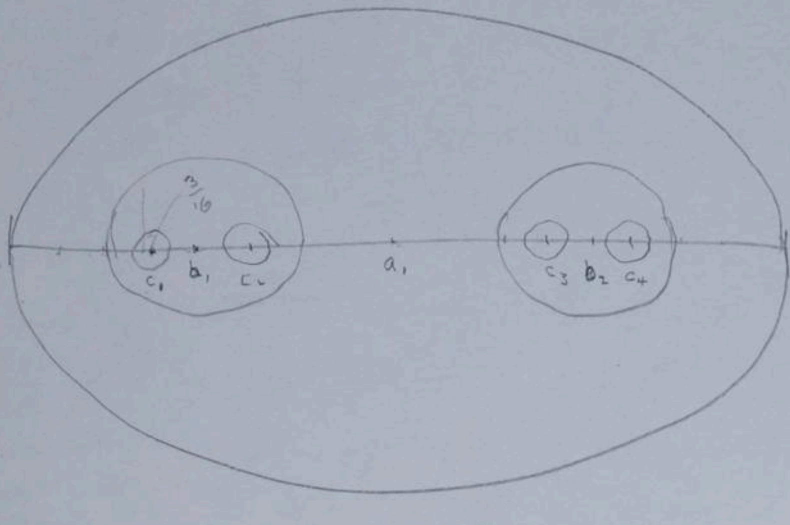


etc.

* the ends of the gaps are always segments of the original/scratched out curve.






3/16
c1
b1
c2
a1
c3
b2
c4

1)* rational (ratio of 2 whole #) numbers
are either terminating decimals
or ∞ decimals infinitely repeating.
All others are irrational

2) rational #s can be counted

3) irrational numbers are uncountable

All the above true in any integral base

$$x + x^2 + x^3 + \cdots = \frac{x}{1-x}$$

---

* or better: all rational numbers
have an infinite expansion in non-
zeros, always ending in ∞
repeating & vice versa

Cantor set has ∞ many topological
equiv, for nearly every one of which the
Cantor set itself is a simple frame
of its properties.

Extension of Cantor sets.

Take out any interior open interval of each remaining closed interval, with proviso that sum of lengths of excluded intervals $\to 1$.

? If sum of lengths $\not\to 1$ resulting set $\cong$ a set containing whole intervals counted as points.

$.ab\cdots p\,p\,p\,p\,p$

$= .ab\cdots p + \frac{p}{10^{p+n}} + \frac{p}{10^{p+2n}} + \cdots$

$= .ab\cdots p + \frac{p}{10^{p}}\left(\frac{1}{10^{n}} + \frac{1}{10^{2n}} + \cdots \right)$

$= .ab\cdots p + \frac{p}{10^{p}}\left(\alpha + \alpha^{2} + \cdots \right)$

$= .ab\cdots p + \frac{p}{10^{p}} \cdot \frac{\alpha}{1-\alpha} = \text{rat.}$

$1-x\,)\,x\,(\,x+x^2$

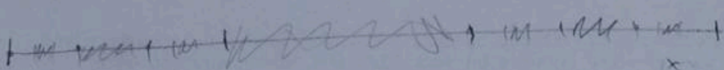
$x-x^2$

x^2

x^2-x^3

x^3

$.0022222200202 00 \cdots\cdots$



$\frac{x}{1-x} = x + x^2 + \cdots + \frac{x^n}{1-x}$

$.1111\cdots\cdots \qquad \frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \cdots + \frac{1}{10^n}$

$\frac{x}{1-x} - (x + x^2 + x^3 + \cdots + x^n)$

$= x + x^2 + \cdots + \frac{x^n}{1-x} - x - x^2 - x^3 - \cdots - x^{n-1}$

$= \frac{x^n}{1-x} - x^n = \frac{x^n - x^n + x^{n+1}}{1-x} = \frac{x^{n+1}}{1-x} = \left(\frac{1}{1-x}\right) x^{n+1}$

$0 \quad 1 \quad \frac{1}{3} \quad \frac{2}{3} \quad \frac{1}{9} \quad \frac{2}{9} \quad \frac{7}{9} \quad \frac{8}{9}$

$$\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \frac{8}{81} + \frac{16}{243} + \frac{32}{729} + \cdots$$

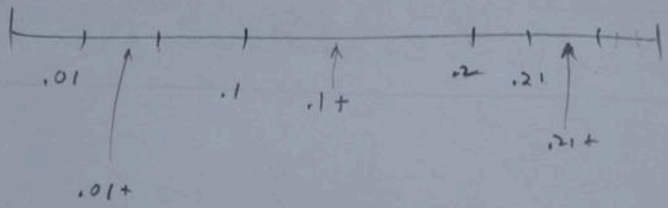
$$= \frac{1}{3}\left(1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \frac{32}{243} + \cdots\right.$$

$$= \frac{1}{3}\left(1 + \left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^4 + \left(\frac{2}{3}\right)^5 + \cdots\right.$$

$$= \frac{1}{3}\left(1 + x + x^2 + x^3 + \cdots\right.$$

$$= \frac{1}{3}\left(1 + \frac{x}{1-x}\right) = \frac{1}{3}\left(1 + \frac{\frac{2}{3}}{\frac{1}{3}}\right) = \frac{1}{3}(1+2)$$

$$= 1$$



Since only those are discarded which have expansion containing a digit 1 followed somewhere by a non-zero digit, but never by an ∞ unbroken series of 2's, we retain all those containing only digits 0 & 2, or a final 1 which can be replaced by an ∞ of 2. Hence we retain just those points expressible by 0's & 2's, These have cardinality of the conts

following the
third point.

endpts can be written without 1

As so written they are all made up of 0's + 2's

with repeating 0 or repeating 2, $\to \infty$

0
.1
.2
1.0
.01
.02
.21
.22
.001
.002
.021
.022
.201
.202
.221
.222

Any sequence of 0 or 2
repeating 0 or 2 $\to \infty$
is an endpt. ?

What about the non repeating

.020020002000002

$a = \frac{1}{2}$

$b_1 = \frac{1}{4}$

$b_2 = \frac{3}{4}$

$c_1 = \frac{3}{16}$

$c_2 = \frac{5}{16}$

$c_3 = \frac{11}{16}$

$c_4 = \frac{13}{16}$

$a_1\ b_1\ c_1\ d_1\ e_1 \ldots$

$\frac{1}{2}\quad \frac{1}{4}\quad \frac{3}{16}\quad \frac{11}{64}$

$\frac{1}{2}\ \left(\frac{2}{4} - \frac{1}{4}\right)\ \left(\frac{4}{16} - \frac{1}{16}\right)\ \left(\frac{12}{64} - \frac{1}{64}\right)$

$a_1 : \frac{2}{2} - \frac{1}{2} = \frac{1}{2}$

$b_1 : \frac{2}{4} - \frac{1}{4} = \frac{1}{4}$

$c_1 : \frac{4}{16} - \frac{1}{16} = \frac{3}{16}$

$d_1 : \frac{12}{64} - \frac{1}{64} = \frac{11}{64}$

$e_1 = \frac{44}{256} - \frac{1}{256} = \frac{43}{256}$

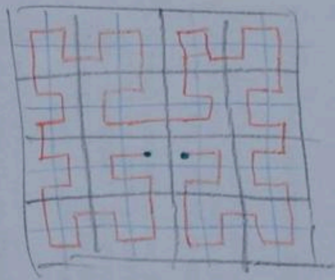
$f_1 = \frac{172}{1024} - \frac{1}{1024}$

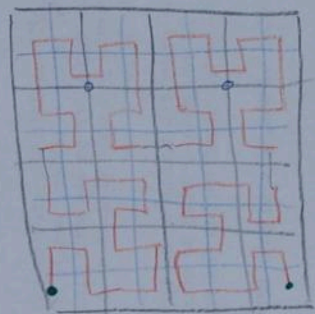
$a_1 > b_1 > c_1 > d_1 > \cdots >$

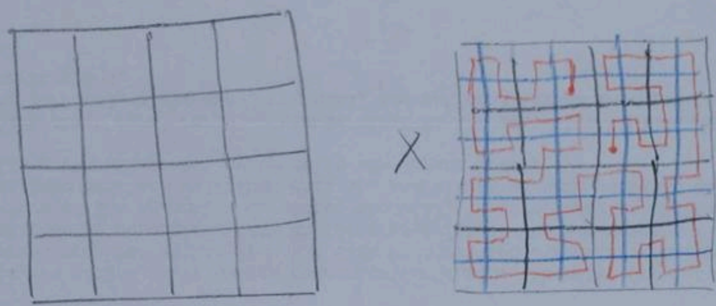
inf decr. seq bdd below by a pos. #

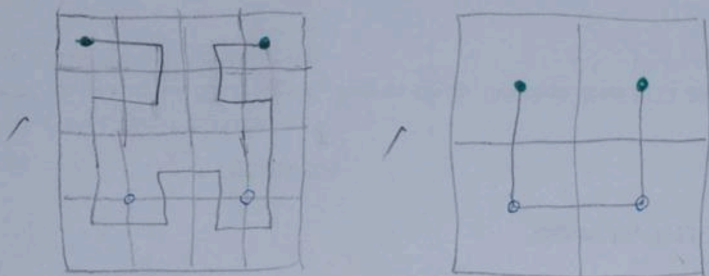
$n_1 = \frac{\quad}{4^{n-1}} - \frac{1}{4^{n-1}}$

An infinite sequence $\alpha_1, \alpha_2, \alpha_3 \cdots > 0$ bd with $\alpha_n - \alpha_{n-1} \to 0$ has a limit









Calculations

B original loan amount (new unpd balance underlined in red on due date [at face]) (at begin of each mo underlined red)

r monthly interest rate

p fixed monthly payment

Jan. 1 0 day $\quad \underline{B}$

1st int $\quad Br \qquad$ 1st decrease $\quad p - Br$

Feb. 1 $\quad B - (p - Br)$

$B + Br - p$

$\underline{B(1+r) - p}$

2d int. $\quad r[B(1+r) - p] \qquad$ 2d decr. $\quad p - r[B(1+r) - p]$

$B(1+r) - p - p + rB(1+r) - rp$

Mar. 1 $\quad = \underline{B(1+r)^2 - 2p - rp}$

3d int $\quad r[B(1+r)^2 - 2p - rp]$

3d decr. $\quad p - r[B(1+r)^2 - 2p - rp]$

$B(1+r)^2 - 2p - rp - p + r[B(1+r)^2 - 2p - rp]$

$= B(1+r)^2 + rB(1+r)^2 - 3p - 3rp - r^2 p$

$= \underline{B(1+r)^3 - 3p - 3rp - r^2 p}$

$= B(1+r)^3 - \frac{p}{r}(3r + 3r^2 + r^3) = B(1+r)^3 - \frac{p}{r}(1 + 3r + 3r^2 + r^3 - 1)$

$= B(1+r)^3 + \frac{p}{r} - \frac{p}{r}(1+r)^3 = \left(B - \frac{p}{r}\right)(1+r)^3 + \frac{p}{r}$

$r\left[B(1+r)^3 - 3p - 3rp - r^2p\right]$

$p - r\left[B(1+r)^3 - 3p - 3rp - r^2p\right]$

$B(1+r)^3 - 3p - 3rp - r^2p - p + r\left[B(1+r)^3 - 3p - 3rp - r^2p\right]$

$B(1+r)^4 - 4p - 6rp - 4r^2p - r^3p$

$= B(1+r)^4 - \left[4p + 6rp + 4r^2p + r^3p\right]$

$= B(1+r)^4 - \frac{p}{r}\left[4r + 6r^2 + 4r^3 + r^4 + 1 - 1\right]$

$= B(1+r)^4 + \frac{p}{r} - \frac{p}{r}\left[1+r\right]^4$

$= \left(B - \frac{p}{r}\right)(1+r)^{n-1} + \frac{p}{r}$ = new unpd bal. at beg. of nth month

$= (-4000)(1+r)^n + 5000$

$$B(1+r)^3 + \frac{k}{r} - \frac{k}{r}(1+r)^3$$

$$= \left(B - \frac{k}{r}\right)(1+r)^3 + \frac{k}{r}$$

$$= \left(1000 - \frac{50}{.01}\right)(1 + .03 + .0003 + .000001) + \frac{50}{.01}$$

$$= (1000 - 5000)(1.030301) + 5000$$

$$= -4000(1.030301) + 5000$$

$$\begin{array}{r} 5000.000000 \\ 4121.204 \\ \hline 878.796 \end{array}$$ ✓ by formula same as straight comp.

$$\begin{array}{r} 878.796 \\ 41.21204 \\ \hline 837.58396 \end{array}$$ st. comp.

$50 - 8.78796 = 41.21204$

$$(-4000)(1.01)^4 + 5000$$

$$= -4000(1.04060401) + 5000$$

By formula

$$= 1000 - 162.41604$$

$$\begin{array}{r} 1000 \\ 162.41604 \\ \hline 837.58396 \end{array}$$ ✓

1/1 B

2/1 int i_1 pay p $i_1 = r B$

bal. $B - (p - i_1)$

int i_2 pay p $i_2 = r[B - (p - i_1)]$

3/1 bal $B - (p - i_1) - (p - i_2)$

int i_3 pay p $i_3 = r[B - 2p + i_1 + i_2]$

4/1 $B - (p - i_1) - (p - i_2) - (p - i_3)$

i_4 $i_4 = r[B - 3p + i_1 + i_2 + i_3]$

5/1 $B - (p - i_1) - (p - i_2) - (p - i_3) - (p - i_4)$

$$i_4 = r\left[B - 3p + i_1 + i_2 + i_3\right]$$

$$i_4' = r\left[B - 2p + i_1 + i_2 + i_3\right]$$

$$i_4' - i_4 = r\left[p\right]$$

Suppose no pay on 4/1 but payment ok on 5/1

1/1 $\quad B$

$\qquad i_1$

2/1 $\quad B - (p - i_1)$

$\qquad i_2$

3/1 $\quad B - (p - i_1) - (p - i_2)$

$\qquad i_3$

4/1 $\quad B - (p - i_1) - (p - i_2) + i_3$

$\qquad i_4'$

5/1 α) $B - (p - i_1) - (p - i_2) + i_3 - (p - i_4')$

if i_3 is not considered 'repaid' —

but

β) $B - (p - i_1) - (p - i_2) + i_3 - i_3 - (p - i_3 - i_4')$

if payment on 5/1 is distr. to cancel i_3.

$\alpha = B - 3p + i_1 + i_2 + i_3 + i_4'$

$\beta = B - 3p + i_1 + i_2 + i_3 + i_4'$

$B = 1000$ $\quad r = .01$ $\quad p = 50$

1000	10	40
− 40		
960	9.60	40.40
− 40.40		
919.600	9.196	40.804
− 40.804		
878.796		

~~$1000(2)^3 - 3.50 + 2.50 \cdot 1$~~

$$1000(1.01)^3 - 3(50) + 2(50)(.01)$$

$$= 1000\left(1 + 3(.01) + 3(.0001) + .000001\right) - 150 + 1$$

$$= 1000 + 30 + .30 + .0010 - 150 + 1$$

30.3010

− 149.

821

851 ~~.3.3010~~

+ 30.3010

~~819.699~~

~~820.000~~

881.3010

1,020.4

50

.01

1000 $\quad$ 1.01

1010

50

960

.0301

$(1.01)^2$

$(1 + .02 + .0004)$

1000

20.40

1020.4

−.50

1019.90

−1.00

919.90

$\cancel{B} - \cancel{p} + \cancel{rB}$

$-\cancel{p} - \cancel{rB} - r^2 B + rp$

$B - r^2 B + rp$

$-p + rB - r^3 B + r^2 p$

$= B(1-r^2) + Br(1-r^2) + r^2 p + rp - p$

$B \qquad rB \qquad p - rB$

$B - p + rB$

$= B(1+r) - p \quad \Big\} \quad r[B(1+r) - p] \qquad p - r[B(1+r) - p]$

$B(1+r) - p - p + rB(1+r) - pr$

$= (1+r)(B + rB) - 2p - pr$

$= B(1+r)^2 - 2p - pr \qquad p - r[B(1+r)^2 - 2p - pr]$

$B(1+r)^2 - 2p - pr - p + r[B(1+r)^2 - 2p - pr]$

$\cancel{B(1+r)^2 - 4p + rB(1+r)^2}$

$B(1+r)^2 - 3p - 3pr + rB(1+r)^2 \cancel{- 2pr} - pr^2$

$B(1+r)^3 - 3p(1+r) - pr^2$

$- p(3 + 3r + r^2)$

unpaid balance in red $\qquad$ r = monthly rate of interest

Ja 1. $\quad B$ (orig. loan) $\qquad$ int $\qquad$ p = fixed pay per month

$-(p - rB)$ $\qquad\qquad$ rB

Fe. 1 $\quad B - p + rB$ $\qquad\qquad$ $r((1+r)B - p)$

$-(p - r((1+r)B - p))$

Mar. 1 $\quad B - p + rB$

$- p + r((1+r)B - p)$

$= B - r^2 B + rp$ $\qquad\qquad$ $r(B - r^2 B + rp)$

$= B(1 - r^2) + rp$ $\qquad\qquad$ $r^2 B - r^3 B + r^2 p$

$$B(1+r)^n - np + (n-1)pr$$

Light year in Å $\quad Å = 10^{-10}$ m

$$L = (300,000)(1000)\,10^{10} \cdot 60 \cdot 60 \cdot 24 \cdot 365 \text{ Å}$$

$$= 3 \cdot 10^{18} \cdot 10^{2} \cdot 36 \cdot 24 \cdot 365$$

$$= 3 \cdot 10^{20} \cdot (3.14)\,10^{5}$$

$$= 9.42 \cdot 10^{25} \cong 10^{26} \text{ Å}$$

$$\frac{10a+b}{99}$$ $0 \leq a \leq 9,\ 0 \leq b \leq 9$

This covers $.\dot{0}\dot{1} \to .\dot{9}\dot{9}$ omitting $\dot{x}\dot{x}$ ~~repeated~~

Classes		
	$10a+b = 3p$	
	$10a+b = 9p$	1
	$10a+b = 11p$	1
	$10a+b = 33p$	2
	$10a+b = 99p$	6

$10a+b$	$.\dot{1}\dot{0}$ [ ]	$.\dot{1}\dot{0} = \frac{10}{99}$	$\frac{10}{99}$
	$.\dot{1}\dot{1}$ [$= .\dot{1}$]	$.\dot{1}\dot{1} = [.\dot{1} =]\ \frac{11}{99} = \frac{1}{9}$	$\frac{1}{9}$
	$.\dot{1}\dot{2}$	$.\dot{1}\dot{2} = \frac{12}{99} = \frac{4}{33}$	$\frac{4}{33}$
	$.\dot{1}\dot{3}$	$.\dot{1}\dot{3} = \frac{13}{99}$	$\frac{13}{99}$
	$.\dot{1}\dot{4}$	$.\dot{1}\dot{4} = \frac{14}{99}$	$\frac{14}{99}$
	$.\dot{1}\dot{5}$	$.\dot{1}\dot{5} = \frac{15}{99} = \frac{5}{33}$	$\frac{5}{33}$
	$.\dot{1}\dot{6}$	$.\dot{1}\dot{6} = \frac{16}{99}$	$\frac{16}{99}$
	$.\dot{1}\dot{7}$	$.\dot{1}\dot{7} = \frac{17}{99}$	$\frac{17}{99}$
	$.\dot{1}\dot{8}$	$.\dot{1}\dot{8} = \frac{18}{99} = \frac{2}{11}$	$\frac{2}{11}$
	$.\dot{1}\dot{9}$	$.\dot{1}\dot{9} = \frac{19}{99}$	$\frac{19}{99}$

$999999 = 3^3 \cdot 7 \cdot 11 \cdot 13 \cdot 37$

$999999 \div 7 = 3^2 \cdot 11 \cdot 13 \cdot 37 = 142857$ (its multiples by 2–8 have same digits in same cyclic order)

$999\,999 \div 11 = 3^3 \cdot 7 \cdot 13 \cdot 37$

$= 189 \cdot 13 \cdot 37$

$= 90909 = 090909$

$999\,999 \div 13 = 76923 = 076923$

$999999 \div 37 = 027\,027$

1	027 027	a	3
2	054 054	b	3
3	081 081	c	3
4	108 108	c	
5	135 135	d	3
6	162 162	e	3
7	189 189	f	3
8	216 216	e	
9	243 243	h	3
10	270 270	a	
11	297 297	i	3
12	324 324	h	
13	351 351	d	
14	378 378	j	3
15	405 405	b	
16	432 432	h	
17	459 459	l	3
18	486 486	m	3
19	513 513	d	
20	540 540	b	
21	567 567	n	3
22	594 594	l	
23	621 621	e	
24	648 648	m	
25	675675	n	
26	702702	a	
27	729729	i	
28	756756	n	
29	783783	j	
30	810810	c	
31	837837	j	
32	864864	m	
33	891 891	f	
34	918 918	f	
35	945 945	l	
36	972 972	i	
37	999999		

no g, no k

12 cycles

a	ot	1	10	26
b		2	15	20
c		3	4	30
d		5	13	19
e		6	8	23
f		7	33	34
h		9	12	16
i		11	27	36
j		14	29	31
l		17	22	35
m		18	24	32
n		21	25	28

1	076923	a
2	153846	b
3	230769	a'
4	307692	a''
5	384615	b'
6	461538	b''
7	538461	b'''
8	615384	b''''
9	692307	a'''
10	769230	a''''
11	846153	b'''''
12	923076	a'''''
13	999999	

There are 2 cycles

1	09	09	09	a
2	18	18	18	b
3	27	27	27	c
4	36	36	36	d
5	45	45	45	e
6	54	54	54	e
7	63	63	63	d
8	72	72	72	c
9	81	81	81	b
10	90	90	90	a
11	99	99	99	

5 cycles

base 9

$$
\begin{array}{r}
7\,\overline{)\,1.\dot{1}2\dot{5}} \\
\underline{7} \\
20 \\
\underline{15} \\
40 \\
\underline{38} \\
1
\end{array}
$$

$\frac{1}{7} = .\dot{1}2\dot{5}$

$\frac{2}{7} = .\dot{2}5\dot{1}$

$\frac{3}{7} = .\dot{3}7\dot{6}$

$\frac{4}{7} = .\dot{5}1\dot{2}$

$\frac{5}{7} = .\dot{6}3\dot{7}$

$\frac{6}{7} = .\dot{7}6\dot{3}$

the algorithm fails

$\frac{7}{7} = .\dot{8}8\dot{8}$ $\left\{ \frac{8}{7} = 1.\dot{1}2\dot{4} \right.$

base 10

$7\,\overline{)\,1.\dot{1}4285\dot{7}}$

$\frac{1}{7} = .\dot{1}4285\dot{7}$

$\frac{2}{7}\ .\dot{2}8571\dot{4}$

$\frac{3}{7} = .\dot{4}2857\dot{1}$

$\frac{4}{7} = .\dot{5}7142\dot{8}$

$\frac{5}{7} = .\dot{7}1428\dot{5}$

$\frac{6}{7} = .\dot{8}5714\dot{2}$

$\frac{7}{7} = .\dot{9}9999\dot{9} = 1$

$\frac{8}{7} = 1.14285\dot{1}$

the algorithm fails

$3^3 \cdot 11 \cdot 13 \cdot 37$

1 99 $2(10^2-1) = 10^2 + 10(10-1) + (10-2)$

2 198

3 297 $3(10^2-1) = 2 \cdot 10^2 + 10(10-1) + (10-3)$

4 396

5 495

6 594

$n \cdot 99$ $n \cdot 99 = (n-1)10^2 + 10(10-1) + (10-n)$

$11 \cdot 99 = 1089$ $\overset{?}{=} 10 \cdot 10^2 + 10(10-1) + (10-11)$

$1000 + 90 - 1 = 1089$

$25 \cdot 99 = 2475$ $\overset{?}{=} 24 \cdot 10^2 + 10(10-1) + (10-25)$

$= 2400 + 90 - 15 = 2475$

7 693

8 792

9 891

10 990

11 1089

12 1188

13 1287

14 1386

$b = 8$

$$\frac{1}{7} = .\dot{1}$$

$$\frac{2}{7} = .\dot{2}$$

$$\frac{3}{7} = .\dot{3}$$

$$\frac{4}{7} = .\dot{4}$$

$$\frac{5}{7} = .\dot{5}$$

$$\frac{6}{7} = .\dot{6}$$

$b = 9$

$$\frac{1}{7} = .\dot{1}2\dot{5}$$

$$\frac{2}{7} = .\dot{2}5\dot{1}$$

$$\frac{3}{7} = .\dot{3}7\dot{6}$$

$$\frac{4}{7} = .\dot{5}1\dot{2}$$

$$\frac{5}{7} = .\dot{6}3\dot{7}$$

$$\frac{6}{7} = .\dot{7}6\dot{3}$$

We define an infinite decimal as any number with their representation: $.a_1 a_2 a_3 \dots a_n \dots,\ n=1-\infty$.
For simplicity of statement we exclude in our study all those in which $a_1 = 0$.
We define an infinite repeating decimal as an infinite decimal $(a_1 \neq 0)$ in which all digits after some digit occur cyclically in a fixed period length; i.e., following some digit, say a_μ all digits occur in a repetition of identical groups: $\underbrace{a_{\mu+1}\, a_{\mu+2} \cdots a_{\mu+\nu}}\ \underbrace{a_{\mu+\nu+1} \cdots a_{\mu+2\nu}}$
where $a_{\mu+\nu+1} = a_{\mu+1}$, $a_{\mu+2} = a_{\mu+\nu+2}$, etc
ν is the length of the period. For simplicity of statement $a_{\mu+1}$ is understood to be the digit of minimum rank which initiates the period. For example in $.372\,0134\,0134\,0134\,0 \cdots$, $a_{\mu+1}$ is the first appearance of the digit 0, while in $.372\,1034\,1034\,1034, \dots$ $a_{\mu+1}$ is the first appearance of the digit 1; in $.3721000341\,0341 \dots$ $a_{\mu+1}$ is the third appearance of the digit 0, etc.

$b = 10$

$\frac{1}{11} = .\dot{0}\dot{9}$

$\frac{2}{11} = .\dot{1}\dot{8}$

$\frac{3}{11} = .\dot{2}\dot{7}$

$\frac{4}{11} = .\dot{3}\dot{6}$

$\frac{5}{11} = .\dot{4}\dot{5}$

$\frac{6}{11} = .\dot{5}\dot{4}$

$\frac{7}{11} = .\dot{6}\dot{3}$

$\frac{8}{11} = .\dot{7}\dot{2}$

$\frac{9}{11} = .\dot{8}\dot{1}$

$\frac{10}{11} = .\dot{9}\dot{0}$

$\frac{1}{6} = .1\dot{6}$

$\frac{2}{6} = .\dot{3}$

$\frac{3}{6} = .5$

$\frac{4}{6} = .\dot{6}$

$\frac{5}{6} = .8\dot{3}$

$\frac{1}{5} = .2 = .1\dot{9}$

$\frac{2}{5} = .4 = .3\dot{9}$

$\frac{3}{5} = .6 = .5\dot{9}$

$\frac{4}{5} = .8 = .7\dot{9}$

All of the above can be left to application of much simpler definition by discussing what we call periodic decimals, namely those with representation $.a_1 a_2 \ldots a_\mu a_1 a_2 \ldots a_\mu \ldots$, the group $a_1 a_2 \ldots a_\mu$ being repeated infinitely many times, the period being μ. Short representation will be $.\dot{a}_1 \ldots \dot{a}_\mu$. The number $.a_1 \ldots a_\mu$ (the basic period alone) will be denoted by p. Then $0 < p < .bb\ldots = 1$ where b is the maximum digit ~~one less than the~~ in the notation. ~~base of the notation~~. In ordinary decimals $b = 9$. In diadic notation $b = 1$. [To include $p = 0$ would be trivial; while $p = b, \mu = 1$ is likewise trivial — $.999\ldots = 1$ in base 10; $.111\ldots = 1$ in base 2, etc.]

Theorem. Every rational number x, $0 < x < 1$ has a unique representation (in each base) as a periodic decimal.

clarify! Theorem. Every periodic decimal equals a unique rational number x, $0 < x < 1$.

$7\overline{)2}(.\dot{2}8571\dot{4} \quad \div 3 = .095238 \div 3 = .031746$

$7\overline{)4}(.\dot{5}7142\dot{8}$

$7\overline{)6}(\dot{8}5714\dot{2}$

$3\overline{)285714}$

$3\overline{)95238}$

$3\overline{)31746}$

$11\overline{)10582}$

$2\overline{)962}$

$13\overline{)481}$

37

$2\overline{)571428}$

285714

$\downarrow$

37

$2\overline{)857142}$

428571

$\downarrow$

37

$\frac{1}{7}$	$.\dot{1}4285\dot{7}$	$3^3 \cdot 11 \cdot 13 \cdot 37$	3^3	1
$\frac{2}{7}$	$.\dot{2}8571\dot{4}$	$2 \cdot 3^3 \cdot 11 \cdot 13 \cdot 37$	$2 \cdot 3^3$	2
$\frac{3}{7}$	$.\dot{4}28571$	$3^4 \cdot 11 \cdot 13 \cdot 37$	3^4	3
$\frac{4}{7}$	$.\dot{5}7142\dot{8}$	$2^2 \cdot 3^3 \cdot 11 \cdot 13 \cdot 37$	$2^2 \cdot 3^3$	4
$\frac{5}{7}$	$\dot{7}1428\dot{5}$	$3^3 \cdot 5 \cdot 11 \cdot 13 \cdot 37$	$3^3 \cdot 5$	5
$\frac{6}{7}$	$.\dot{8}5714\dot{2}$	$2 \cdot 3^4 \cdot 11 \cdot 13 \cdot 37$	$2 \cdot 3^4$	6

Every rational number has a unique repeating decimal representation.

Conversely, every repeating decimal is the unique decimal representation of a unique rational number.

Consider the repeating period: $a_1 a_2 \ldots a_n$, and let r equal the obvious rational expressed by the digits preceding the first appearance of the repeating period, and let p be the rational value of the first app of the period. The repeating decimal may then be expressed by:

$$r + p + \frac{p}{10^n} + \frac{p}{10^{2n}} + \cdots$$

$$= r + p\left(1 + \frac{1}{10^n} + \frac{1}{10^{2n}} + \cdots\right)$$

$$= r + p\left(\frac{1}{1 - \frac{1}{10^n}}\right)$$

$$= r + p\left(\frac{10^n}{10^n - 1}\right)$$

Ex.

$$.1\dot{2}\dot{3} = .1 + .023\left(\frac{100}{99}\right)$$

$$= .1 + \frac{2.3}{99} = \frac{9.9 + 2.3}{99} = \frac{12.2}{99}$$

$$\begin{array}{r} .1\dot{2}\dot{3} \\ 99\overline{)12.2} \\ 9\,9 \\ \hline 2\,3\,0 \\ 1\,9\,8 \\ \hline 3\,2\,0 \\ 2\,9\,7 \\ \hline 2\,3 \end{array}$$

$$8)\overline{12.1}\;\; 1.5125$$

```
8) 12.1 | 1.5125
    8
   ---
    41
    40
   ---
     10
      8
    ---
     20
     16
    ---
      40
      40
     ---
       0
```

Here $p = 0$

Ex. $.1\dot{2}\dot{4} = .1 + .024\left(\frac{100}{99}\right)$

$$= .1 + \frac{2.4}{99} = \frac{9.9 + 2.4}{99}$$

$$= \frac{3.3 + .8}{33} = \frac{4.1}{33} = \frac{41}{330}$$

? $\quad .1 + \frac{2.3}{98} = \frac{9.8 + 2.3}{9.8} = \frac{12.1}{9.8} = \frac{1210}{980}$

? $\quad .1 + \frac{2.3}{7} = \frac{.7 + 2.3}{7} = \frac{3}{7}$

```
7) 3.0 | .428571
   28
   --
    20
    14
    --
     60
     56
     --
      40
      35
      --
       50
       49
       --
        10
         7
        --
         3
```

Here $r = 0$

$$.\dot{4}2857\dot{1} = 0 + .428571\left(\frac{10^6}{999999}\right)$$

$$= \frac{428571}{999999}$$

7. Every repeating decimal of period length n has a rotation representation in which the denominator is $10^n - 1 = \underbrace{9 \cdots 9}_{n \text{ digits}}$!
??? and this denominator has a factor of which the digits are a permutation of the repeating period ???

$\cdot 999999 \overline{)428571}($

$111111 \overline{)47619}$ $\qquad 11 \overline{)47619}($

$37037 \overline{)15873}(\; .\dot{4}2857\dot{1}$

$$\begin{array}{r}
148148 \\
\hline
105820 \\
74074 \\
\hline
317460 \\
296296 \\
\hline
211640 \\
185185 \\
\hline
264550 \\
259259 \\
\hline
52910 \\
37037 \\
\hline
15873
\end{array}$$

mystery – what happened to $\frac{3}{7}$?

$\frac{3}{7} = .\dot{4}2857\dot{1} = \frac{428571}{999999}$ $\qquad 2999997 = 7 \cdot 428571$!

$2999997 = 3 \cdot 999999$!

142857 $\qquad 3 \overline{)428571}(142857$

142857 $\qquad 7 \overline{)999999}(142857$

..... oh but why is 142857 the common factor?

Because there has to be a common factor for $\frac{1}{7}$ and the rest [illegible]

But why is the common factor a permutation of the period.

for 5/7 6 = 2.3 —

3 | 428571
3 | 142857
3 | 47619
3 | 15873
11 | 5291
13 | 481
37

37!

$7)5\ (.\dot{7}1428\dot{5}$

3) 714285
3) 238095
3 | 79365
5) 26455
11) 5291
13) 481
37

not 57

$7)1\ (.\dot{1}4285\dot{7}$

3 | 142857
⋮
13)
37

not 17

Some of these characters stem from the fact that $\dot{4}2851\dot{7}$ belongs to 7 equally as does any other of the possible 6 sequences i.e. $\dot{2}8517\dot{4}$ $\dot{8}5174\dot{2}$, etc

~~1/7 = .142857~~

~~1/11~~

11) 1 (.0909
99

1

$.\dot{0}\dot{9}$

.18

.27

.36

.45

.54

.63

.72

.81

13) 1 (.0̇7692̇3

91

90

78

120

117

30

26

40

39

10

1	076923
2	153846
3	230769
4	307692
5	384615
6	461538
7	538461
8	615384
9	692307

14) 1 (.07̇14285̇

98

20

14

60

56

40

28

120

112

80

70

100

071425

142850

$\frac{1}{21} = \frac{\dot{1}4285\dot{7}}{3}$

$= .\dot{0}4761\dot{9}$

$\frac{1}{28} = \frac{\dot{1}4285\dot{7}1428}{4}$

$= .035\dot{7}142\dot{8}$

Factors of 9...9

9	3^2
99	$3^2 \cdot 11$
999	$3^3 \cdot 37$
9999	$3^2 \cdot 11 \cdot 101$
99999	$3^2 \cdot 41 \cdot 271$
999999	$3^3 \cdot 7 \cdot 11 \cdot 13 \cdot 37$

Factors of 111111

1	1
11	11
111	$3 \cdot 37$
1111	$11 \cdot 101$
11111	$41 \cdot 271$
111111	$3 \cdot 7 \cdot 11 \cdot 13 \cdot 37$

Let $\Pi(n) = p_1 \cdot p_2 \cdots p_n$

Then $\Pi(n) + x$ is divisible

~~[illegible]~~ for $x = 2, 3, 4, \ldots, p_n$.

1. If x is prime x is one of the factors of $\Pi(n)$; hence of ~~x divides~~ $\Pi(n) + x$

2. If x is not prime let α be any one of its prime factors ~~$\alpha_1, \alpha_2, \ldots \alpha_n$~~

Obviously α is a factor of $\Pi(n)$; ~~and also a factor of x.~~ Hence ~~[illegible]~~ of $\Pi(n) + x$ ~~is divisible by α.~~

Let $n = 1{,}000{,}000$. Then

$\Pi(1{,}000{,}000) + 1$ is prime

$\Pi(1{,}000{,}000) + 2$

$\pi(9) = 223092870$

$\pi(9) + 14$ is divisible

$223092884 \div 2$

$= 111546442 \div 7$

$= 15935103$

$\pi(9) + 16$ is divisible

$223092886 \div 2$

111546443

$\pi(9) + 15$ is divisible

223092885

$\pi(9) + x$

x $\cancel{2}$ $\cancel{3}$ $\cancel{4}$ $\cancel{5}$ $\cancel{6}$ $\cancel{7}$ $\cancel{8}$

$\cancel{9}$ $\cancel{10}$ $\cancel{11}$ $\cancel{12}$ $\cancel{13}$ $\cancel{14}$ $\cancel{15}$

$\cancel{16}$ $\cancel{17}$ $\cancel{18}$ $\cancel{19}$ $\cancel{20}$ $\cancel{21}$ $\cancel{22}$ $\cancel{23}$

The addition of what x will yield a sum not obviously divisible? None!

30030

✓ 30032

✓ 30033

✓ 30034

✓ 30035

✓ 30036 2.3.4.6.12 2,3

✓ 30037 4291

✓ 30038 300.31

✓ 30039

✓ 30040

$$\begin{array}{r} 180 \\ 180 \\ \hline 32400 \end{array}$$

✓ 30041

✓ 30042

✓ 30043

$$\Pi(n) = p_1 \cdot p_2 \cdot p_3 \cdots p_n$$

2	$\Pi(1) = 2$
3	$\Pi(2) = 6$
5	$\Pi(3) = 30$
7	$\Pi(4) = 210$
11	$\Pi(5) = 2310$
13	$\Pi(6) = 30030$
17	$\Pi(7) = 510510$
19	$\Pi(8) = 9699690$
23	$\Pi(9) = 223092870$

$(p_1 \cdot p_2 \cdot p_3 \cdots p_n) + q$ $2 \leq q \leq p_n$ (i.e. $q = 2, 3, \ldots, p_n$)

```
 210
 210
2310
```

2312 ✓
— 2313 ✓
2314 ✓
2315 ✓
— 2316 ✓
2317
2318 ✓
— 2319 ✓
2320 ✓
2321

$(p_1 \cdot \cdots p_{1,000,000})$

$\frac{1}{2}$ of the multiples of 3 are multiples of 2

$\frac{1}{3}$ of the " " 5 " " 3

$\frac{1}{5}$ " " 7 " 5

n $\frac{n}{2}$ $\frac{n}{4}$

1 2 3 4 5 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30
31 32 33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48 49 50

8 9 10
14 15 16
20 21 22
24 25 26 27 28
32 33 34 35 36
38 39 40
44 45 46
48 49 50 51 52
54 55 56
62 63 64 65 66

74 75 76 77 78
80 81 82
84 85 86 87 88
90 91 92 93 94 95 96
98 99 100
104 105 106
110 111 112
114 115 116 117 118 119 120

Base 12   1 2 3 4 5 6 7 8 9 U | ∩ 10

11 12 13 14 15 16 17 18 | 19 1U 1∩ 20

21 22 23 24 25 26 | 27 28 29 2U 2∩ 30

2 3 5 7 ∩ 11 15 17 1∩ 25 27

10) $369 = 2(12^2) + 6(12) + 9$

$$\begin{array}{r} 369 \\ -\,288 \\ \hline 81 \end{array}$$

12 | 269   15

10) $2640 = 12^3 + 6(12^2) + 4(12)$

$$\begin{array}{r} 2640 \\ -\,1728 \\ \hline 912 \\ -\,864 \\ \hline 48 \end{array}$$

1640   ∩

$3 \overline{)369}$

123   $\left(\overset{(12)}{=} U(12) + 3\right)$

$= U3$

$3 \overline{)269}$

U3

9

Let Π_n be prod of first n primes, $p_1, p_2, \ldots, p_n$

Then Π_n is even. Hence $\Pi_n + 1$ has last digit δ 1, 3, 5, 7 or 9

If $\delta = 5$ $\Pi_n + 1 \neq$ prime and has the prime factor 5

1	1	
10	2	
11	3	
100	4	
101	5	101
110	6	
111	7	
1000	8	
1001	9	
1010	10	
1111	15	1111
10100	20	
11001	25	11001
	55	110111

Numbers to the base 10 are even if and only if the last digit is even.

Odd numbers to the base 10 are divisible by 5 if and only if the last digit is 5

Numbers to the base 2 are even if and only if the last digit is 0.

Odd numbers to the base 2 are divisible by 5 if and only if — ? whatever goes in here it must involve all digits.

An odd number to base 2: $a_n \cdot 2^n + a_{n-1} 2^{n-1} + \cdots + a_2 2 + 1$, where $a_n = 0$ or 1.

i.e. $a_n a_{n-1} \cdots a_2 1$

$$= a_n (3-1)^n + a_{n-1}(3-1)^{n-1} + \cdots + a_2(3-1) + 1$$

$$= 3A_n \cdot a_n + 3A_{n-1} \cdot a_{n-1} + \cdots + 3 \cdot a_2 \pm (a_n - a_{n-1} + \cdots \pm 1)$$

[+ if n is even, − if n odd]

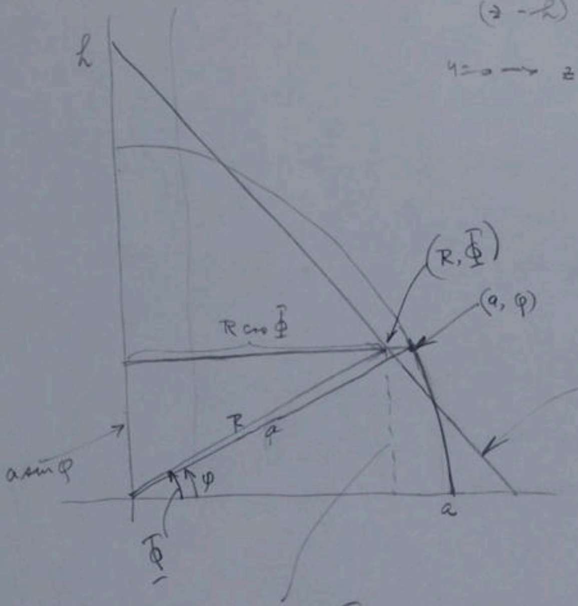
1	3	5	7	9
1	9	5	9	1
	7		3	9
	1		1	
	3		7	

6

$$(z-h)^2 - \alpha(x^2+y^2) = 0$$

$$y=0 \longrightarrow z-h = -\sqrt{\alpha}\, x$$

$$1025 = 10000000001$$



$$z-h = -\sqrt{\alpha}\, x$$

$$R\sin\Phi - h = -\sqrt{\alpha}\, R\cos\Phi$$

$$R\sin\Phi = a\sin\varphi$$

$$\boxed{\begin{aligned} R &= a\sin\varphi \csc\Phi \\ R\sin\Phi - h &= -\sqrt{\alpha}\, R\cos\Phi \end{aligned}}$$

$$a\sin\varphi - h = -\sqrt{\alpha}\; a\sin\varphi \csc\Phi\cos\Phi$$

$$\cot\Phi = \frac{h - a\sin\varphi}{a\sqrt{\alpha}\sin\varphi}$$

$$\sin\Phi = \frac{1}{\sqrt{1 + \dfrac{h^2 - 2ah\sin^2\varphi}{a^2\alpha\sin^2\varphi}}}$$

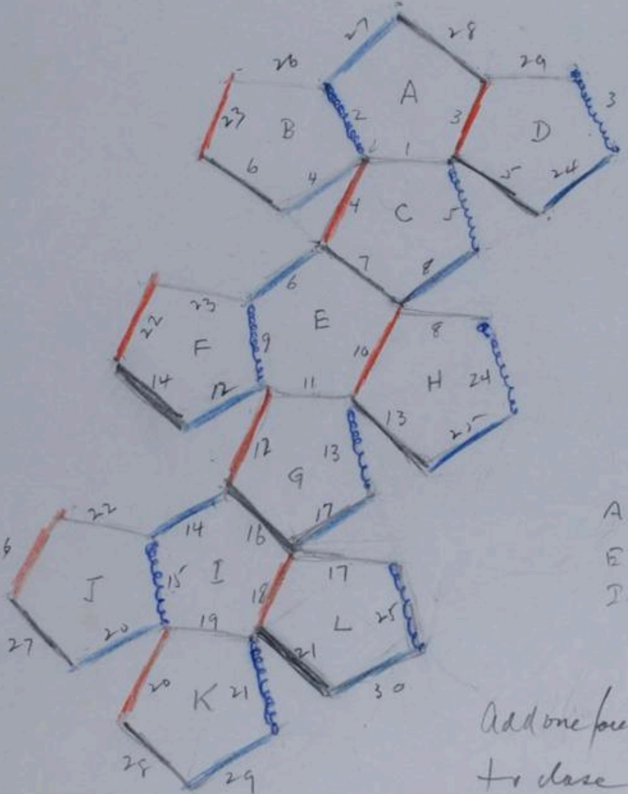
$$= \frac{a\sqrt{\alpha}\sin\varphi}{\sqrt{h^2 + (a^2\alpha - 2ah)\sin^2\varphi}}$$

2/20/71

$V - E + F = 2$

$20 - 30 + 12 = 2$ ✓

Dodecahedrons of pentagonal faces.
Regular ok.
What other kinds are there in which all pentagon angles are equal?



	V	E	F
A-D	14	17	4
E-H	12	16	4
I-L	12	16	4
	38	- 49	+ 12 = 1 ✓

Add one face to close $\quad 38 - 38 + 1 = 1$

$76 - 87 + 13 = 2$

$V - E + F = 2$

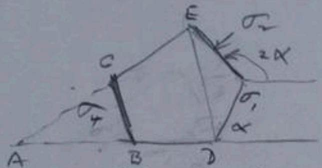
Def. A plane polygon of 5 sides and equal angles shall be called an arp (angle-reg. pentagon)

Th. If 3 sides of an arp are given, the arp is determined.

Obvious.

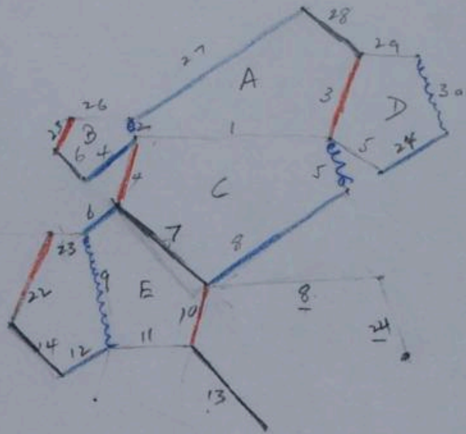
Th. If any 3 sides of an arp are given the arp is determined.

Let the five consecutive sides be denoted by $\sigma_1, \sigma_2, \sigma_3, \sigma_4$ and σ_5 with directions $\alpha, 2\alpha, 3\alpha, 4\alpha, 5\alpha$, $\alpha = 72°$



Without loss of generality the given sides may be σ_1, σ_2 & σ_4. Clearly the length of σ_4 determines σ_3 & σ_5. For the shape of ABC is determined and hence its sides. Also the shape of ADE (not necessarily the same as ABC) is known from σ_1 & σ_2, as is DE. AD - AE being determined as are $\sigma_3 + \sigma_5$.

	Free	Deter.
	1✓	
	2✓	
A	3✓	
	~~27~~	27✓
	~~28~~	28
	4✓	
	6✓	
B	~~23~~	23✓
	~~26~~	26✓
	5✓	
	24✓	
D	~~29~~	29
	~~30~~	30✓
C		7✓
		8✓
E	9✓	
		10✓
		11✓
F	12✓	
		14
		22✓
H		13✓
		25✓
G		16
		17✓
J		20
		15



L

18
21

I can be fitted in only 1

14, 15, 16, 18 are compatible which they need not be

K If I can be fitted in K will fit only if 19, 20, 21, 28, 29 are compatible — no reason to see that they will be

As soon as 3 consecutive sides are given the other 2 are determined

Can we say any 3 sides Yes

At the time of making the last choice,
side 12 (in constituting F)

H is already determined

There are 2 roads to 15

1) deter. by J (26, 27 + 22 from F)

2) deter by I (14, 16, 18)

where 14 from F, 16 from G + 18 from L via G. (17)

Fixing 12 deter. 14, 22 ; 16, 17

22, 26, 27 → J + 15

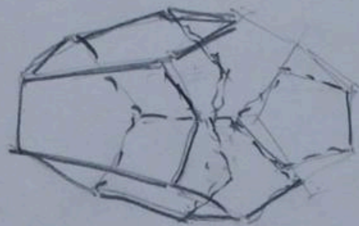
17, 25, 30 → L → 18

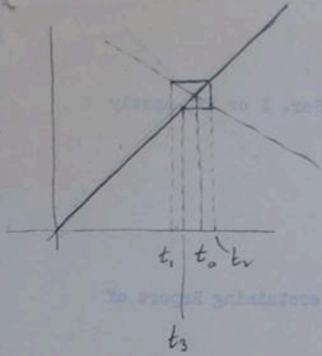
+ 14, 16, 18 → I + 15

If we can get same value of 15 by the 2 paths then 19 is unique.

But will 19 with 20 + 21 yield either 28 or 29.

as assumed by A + J. ???





Let $t = f(t)$ define a soln t_0
where $f'(t)$ is cont $|t - t_0| < \epsilon$.
Then if $f'(t_0) < 0$ and $|f'(t_0)| < 1$

and if $|t_1 - t_0|$ is sufficiently
small but positive the
sequence $t_1, t_2, t_3, \ldots \to t_0$

where $t_2 = f(t_1)$, $t_3 = f(t_2)$, ...

In particular, if $t_1 < t_0$ then

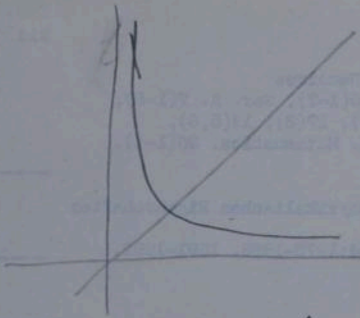
$t_1 < t_3 < t_5 \ldots < t_0$

and $t_2 > t_4 > t_6 > \ldots > t_0$.

and $|t_{n+1} - t_n| \to 0$

7. If $|f'(t_0)| > 1$ consider the eqn $\varphi(t) = f^{-1}(t) = t$

then $t = \varphi(t)$ has the root t_0 and with conditions
on $\varphi(t)$ equivalent to those above a similar
theorem holds.



$$t = \frac{1}{t^2} \left(\equiv f(t) \right)$$

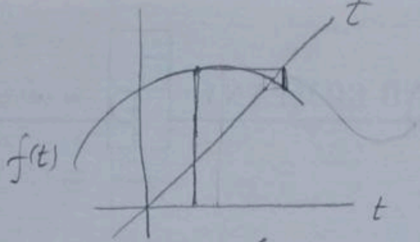
$$t_0 = 1 \qquad f'(t_0) = -2$$

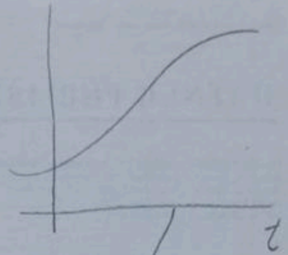
Conditions are not satisfied.

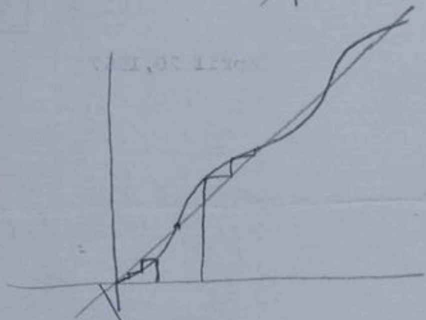
Consider now $\varphi(t) = \frac{1}{\sqrt{t}}$

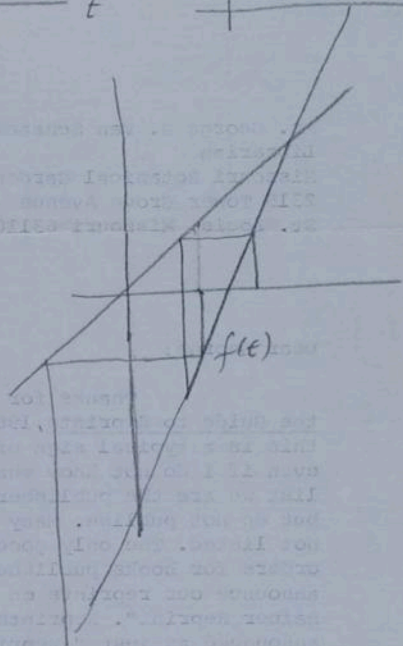
For $t = \frac{1}{\sqrt{t}}$ the conditions are satisfied.

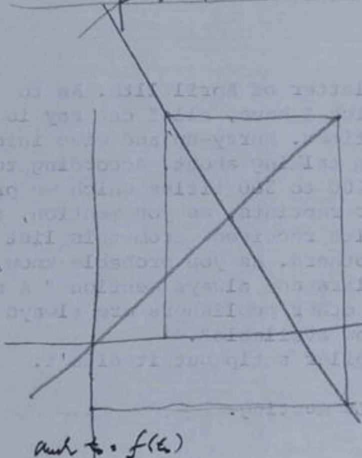
$t = f(t)$







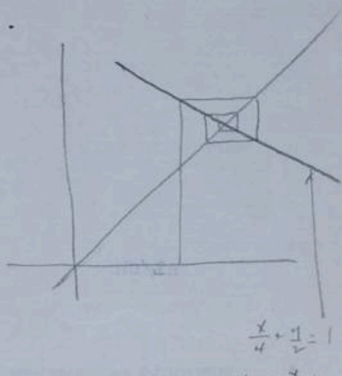




and $t_0 = f(t_0)$

Let $t_1 = f(t)$; $t_2 = f(t_1)$, $t_3 = f(t_2)$ ...

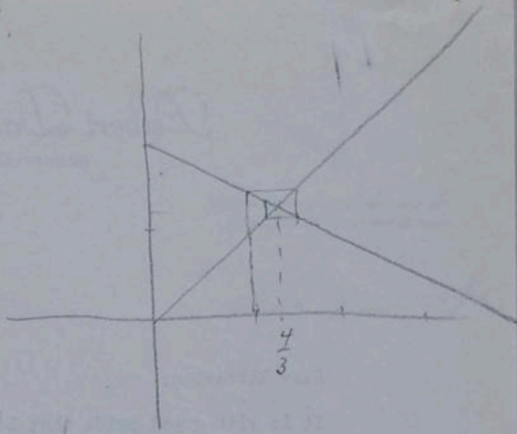
When will $t_1, t_2, \ldots$ converge to t_0?



$\frac{x}{4} + \frac{y}{2} = 1$

$y = -\frac{x}{2} + 2$

$t = -\frac{t}{2} + 2 \longrightarrow t = \frac{4}{3}$



$t_1 = 1 \qquad = 1$

$t_2 = 2 - \frac{1}{2} = \frac{3}{2}$

$t_3 = 2 - \frac{3}{4} = \frac{5}{4}$

$t_4 = 2 - \frac{5}{8} = \frac{11}{8}$

$t_5 = 2 - \frac{11}{16} = \frac{21}{16}$

$t_6 = 2 - \frac{21}{32} = \frac{43}{32}$

$t_7 = 2 - \frac{43}{64} = \frac{85}{64}$

$t_8 = 2 - \frac{85}{128} = \frac{171}{128}$

	1	2	3	4	5	6	7	8	
u_n / 2^{n-1}	1,	3 / 2,	5 / 4,	11 / 8,	21,	43,	85,	171,	341

u_{2p+1} 1, 5, 21, 85, 341

u_{2p} 3, 11, 43, 171, ...

$u_{2p+1} = 4u_{2p-1} + 1$

$u_{2p} = 4u_{2p-2} - 1$

$u_2 = 4u_0 - 1$ $\qquad$ Set $u_0 = 1$

$u_4 = 4u_2 - 1$

$= 4(4u_0 - 1) - 1$

$= 4^2 u_0 - 4 - 1$

$u_6 = 4(4^2 u_0 - 4 - 1) - 1$

$= 4^3 u_0 - 4^2 - 4 - 1$

$u_{2p} = 4^p - 4^{p-1} - 4^{p-2} - \cdots - 4^0$

$= 4^p - (4^0 + 4 + 4^2 + \cdots + 4^{p-1})$

$= 4^p - \frac{1-4^p}{1-4} = \frac{4^p - 4^{p+1} - 1 + 4^p}{-3}$

$\frac{u_{2p}}{2^{2p-1}} = \frac{4^{p+1} - 2\cdot 4^p + 1}{3\cdot 2^{2p-1}} = \frac{2(4^{p+1} - 2\cdot 4^p + 1)}{3\cdot 4^p} = \frac{\left(4 - 2 + \frac{1}{4^p}\right)}{3} \rightarrow \frac{2}{3}$

$1 + a + a^2 + \cdots + a^{p-1}$

$= \frac{1-a^p}{1-a}$

$= \frac{4^{p+1} - 2\cdot 4^p + 1}{3}$

What is amount of 1, 2, 3, ..., n @ i/per.

$$A_1 = 1+i$$

$$A_2 = (1+i)(1+i) + 2(1+i)$$

$$A_3 = (1+i)^3 + 2(1+i)^2 + 3(1+i)$$

$$A_n = (1+i)^n + 2(1+i)^{n-1} + 3(1+i)^{n-2} + \cdots + n(1+i)$$

$$= (1+i)^n + (1+i)^{n-1} + \cdots + (1+i)$$

$$(1+i)^{n-1} + \cdots + (1+i)$$

$$(1+i)^{n-2} + \cdots + (1+i)$$

$$- \; - \; -$$

$$= (1+i)\frac{((1+i)^n - 1)}{i}$$

$$+ (1+i)\frac{((1+i)^{n-1} - 1)}{i}$$

$$- \; - \; -$$

$$+ (1+i)$$

$$= (1+i)\left[s_{\overline{n}|} + s_{\overline{n-1}|} + \cdots + s_{\overline{1}|} \right]$$

Representation of N in terms of successive divisions & associated remainders

$$N_1 = a_1 b_1 + \alpha_1$$

$$b_1 = a_2 b_2 + \alpha_2$$

$$b_2 = a_3 b_3 + \alpha_3$$

$$N = a_1\left[a_2\left[a_3 b_3 + \alpha_3\right] + \alpha_2\right] + \alpha_1$$

$$= a_1 a_2 a_3 b_3 + a_1 a_2 \alpha_3 + a_1 \alpha_2 + \alpha_1$$

$$N = a_1 \cdots a_n b_n + a_1 \cdots a_{n-1}\alpha_n$$

$$+ \cdots + a_1 \alpha_2 + \alpha_1$$

If $n=1$ the formula says

$$N = a_1 b_1 + \alpha_1 \checkmark$$

If $n=2$

$$N = a_1 a_2 b_2 + a_1 \alpha_2 + \alpha_1$$

If $n=3$

$$N = a_1 a_2 a_3 b_3 + a_1 a_2 \alpha_3 + a_1 \alpha_2 + \alpha_1$$

$$N(n) = a_1 \cdots a_n b_n \overbrace{+ a_1 \cdots a_{n-1}\alpha_n + \cdots + a_1 \alpha_2 + \alpha_1}^{R}$$

$$N(n+1) = a_1 \cdots a_{n+1} b_{n+1} + a_1 \cdots a_n \alpha_{n+1}$$

$$+ a_1 \cdots a_{n-1}\alpha_n + \cdots + a_1 \alpha_2 + \alpha_1$$

now $b_n = a_{n+1} b_{n+1} + \alpha_{n+1}$

$$N(n+1) = a_1 \cdots a_n\left(a_{n+1} b_{n+1} + \alpha_n\right) + R$$

$$= a_1 \cdots a_{n+1} b_{n+1} + a_1 \cdots a_n \alpha_{n+1} + R$$

$\checkmark$

$$N = A_n b_n + A_{n-1}\alpha_n + A_{n-2}\alpha_{n-1}$$

$$+ \cdots a_1 \alpha_2 + \alpha_1$$

Set $a_i = 2$, then $A_i = 2^i$

$$N = 2^n \cdot b_n + 2^{n-1}\alpha_n + 2^{n-2}\alpha_{n-1} + \cdots + 2^0 \cdot \alpha_1$$

To select the minimum no. of integral weights which by composition will give all weights

$1, 2, \ldots, 2^n - 1$

One set is clearly $1, 2, 4, \ldots, 2^{n-1}$, of n pieces.

This is minimum and unique.

To begin with, no set with $n-1$ or fewer can satisfy.

For to obtain $2^n - 1$ values by the type of composition proposed requires $\not< n$ pieces, since $n-1$ pieces have fewer than $2^n - 1$ count, namely

$$a_1, \ldots, a_{n-1};\ a_1 + a_2,\ a_1 + a_3\ \ldots,\ a_1 + a_{n-1},$$

etc: how, what is count.

$$(n-1) + \frac{(n-1)(n-2)}{2} + \frac{(n-1)(n-2)(n-3)}{3!} + \ldots + \frac{(n-1)\cdots 1}{(n-1)!}$$

$$n-(n-1)$$

$$= 2^{n-1} - 1 < 2^n - 1$$

$$\frac{(n-1)\cdots 1}{(n-1)!} = 1$$

$$\frac{(n-1)(n-2)\cdots 2}{(n-2)!} = n-1$$

$$\frac{(n-1)(n-2)(n-3)\cdots 4}{(n-4)!} \cdot \frac{3.2.1}{3.2.1} = \frac{(n-1)(n-2)(n-3)}{3!}$$

$$\frac{(n-1)(n-2)\cdots 3}{(n-3)!} = \frac{(n-1)(n-2)(n-3)!}{(n-3)!\,2} = \frac{(n-1)(n-2)}{2}$$

$$(n-1) + \frac{(n-1)(n-2)}{2!} + \frac{(n-1)(n-2)(n-3)}{3!} + \cdots + \frac{(n-1)\cdots 1}{(n-1)!}$$

$$= 1 + (n-1) + \frac{(n-1)(n-2)}{2!} + \frac{(n-1)(n-2)(n-3)}{3!} + \cdots (n-1)$$

$$(1+1)^n = 1 + n + \frac{n(n-1)}{1.2} + \frac{n(n-1)(n-2)}{1.2.3}$$

$$+ \cdots + \frac{n\cdot(n-1)\cdots(n-(n-2))}{(n-1)!} + \frac{n\cdot(n-1)\cdots(n-(n-1))}{n!}$$

$$2^{n-1} = 1 + (n-1) + \frac{(n-1)(n-2)}{2!} + \frac{(n-1)(n-2)(n-3)}{3!} + \cdots$$

$$+ \frac{(n-1)(n-2)\cdots 1}{(n-1)!}$$

circle

$$\frac{\pi r^2}{4r^2} = \frac{\pi}{4}$$

sphere

$$\frac{\frac{4}{3}\pi r^3}{8r^3} = \frac{\pi}{6}$$

$$\iiiint_0^{\sqrt{r^2-y^2-z^2-t^2}} dx\,dy\,dz\,dt \qquad x^2+y^2+z^2+t^2 = r^2$$

$$\iiint_0^{\sqrt{r^2-z^2-t^2}} \sqrt{r^2-y^2-z^2-t^2}\,dy\,dz\,dt$$

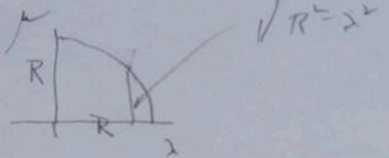
$$= \int_0^r \int_0^{\sqrt{r^2-t^2}} \frac{\pi}{4}\left(r^2-z^2-t^2\right) dz\,dt = \frac{\pi}{4}\int_0^r \left[r^2 z - \frac{z^3}{3} - t^2 z\right]_0^{\sqrt{r^2-t^2}} dt$$

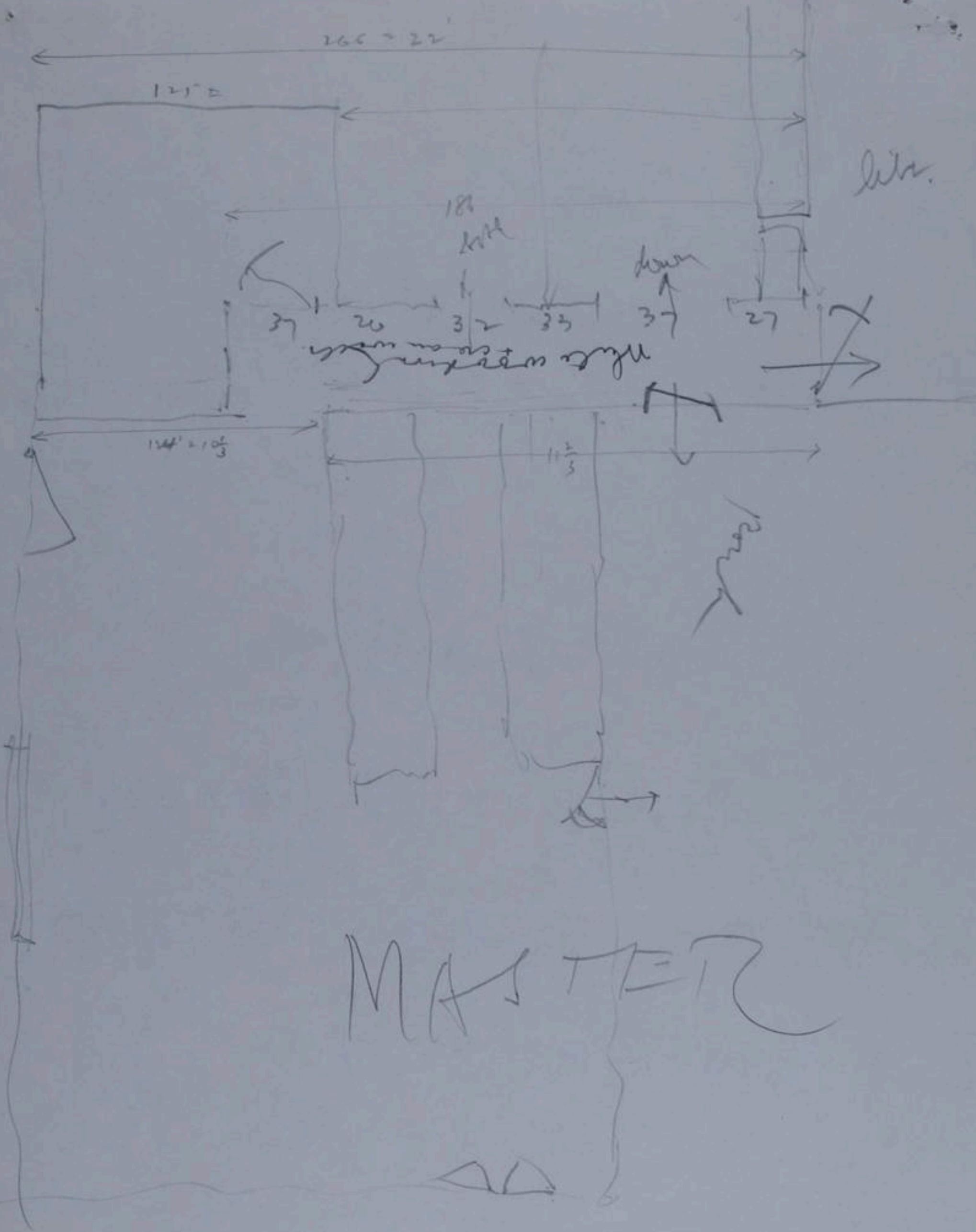
$$= \frac{\pi}{4}\int_0^r \left(r^2\sqrt{r^2-t^2} - \frac{(r^2-t^2)\sqrt{r^2-t^2}}{3} - t^2\sqrt{r^2-t^2}\right) dt$$

$$= \frac{\pi}{4}\int_0^r \left(\frac{2}{3} r^2\sqrt{r^2-t^2} + \frac{2}{3} t^2\sqrt{r^2-t^2}\right) dt$$

$$\int_0^R \sqrt{R^2-\lambda^2}\,d\lambda$$

$$= \frac{\pi}{4} R^2$$





MASTER

$$\sum = \frac{\pi}{4}\cdot\frac{2}{3}\left[\frac{\pi}{4}r^4 - \frac{\pi r^4}{16}\right]$$

$$= \frac{\pi}{4}\cdot\frac{2}{3}\cdot\frac{3}{16}\,\pi r^4 = \frac{\pi^2 r^4}{32}$$

total vol. of hyper sphere $= \frac{\pi^2 r^4}{2} = 5$

cube $= 16r^4 = 16$

ratio $= \dfrac{\frac{\pi^2 r^4}{2}}{16 r^4} = \frac{\pi^2}{32}$ $\qquad \frac{\pi^2}{128}$

.2246 [$1 = 1.000$

$\frac{\pi}{4} = .7854$

.2618

.0456 [$\frac{\pi}{6} = .5236$

.2152

$\frac{\pi^2}{32} = .3084$

sphere $\frac{4}{3}\pi r^3$

cyl $2\pi r^3$

cube $8r^3$

$\frac{1}{4}\pi = \frac{\pi}{4}$

$\frac{1}{6}\pi = \frac{\pi}{6}$

$\frac{\pi}{32}\pi = \frac{\pi}{10}$

3.14159
3.1416
18 8496
31 416
125664
3 1416
94248
32) 9.8696 (.3084
96
269
256
13 9

$$\int_0^r r^2\sqrt{r^2-t^2}\,dt = r^2\int_0^r \sqrt{r^2-t^2}\,dt = \frac{\pi}{4}r^4$$

$$\int_0^r t^2\sqrt{r^2-t^2}\,dt \qquad t = r\sin\theta$$

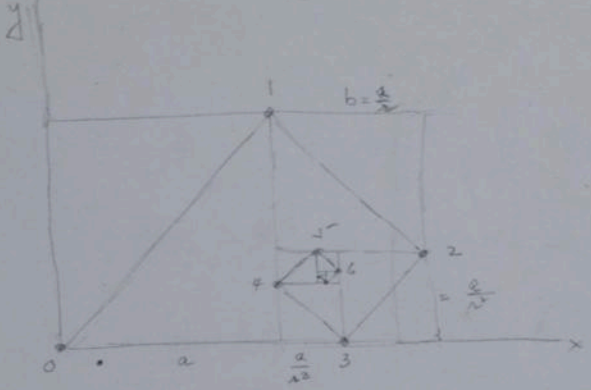
$$= \int_0^{\frac{\pi}{2}} r^2\sin^2\theta\cdot r\cos\theta\cdot r\cos\theta\,d\theta$$

$$= r^4\int_0^{\frac{\pi}{2}}\sin^2\theta\cdot\cos^2\theta\,d\theta = \frac{r^4}{8}\int_0^{\frac{\pi}{2}}\sin^2 2\theta\,d2\theta$$

$$= \frac{r^4}{16}\int_0^{\frac{\pi}{2}}\frac{1-\cos 4\theta}{2}\,d4\theta = \frac{r^4}{32}\Big[4\theta - \sin 4\theta\Big]_0^{\frac{\pi}{2}}$$

$$= \frac{r^4}{32}\cdot 2\pi = \frac{\pi r^4}{16}$$

Logarithmische Spiralen



$$\frac{b}{a} = \frac{a}{a+b} \longrightarrow a^2 - ab - b^2 = 0 \longrightarrow a = \frac{b \pm \sqrt{b^2 + 4b^2}}{2}$$

$$\frac{a}{b} = \frac{1}{2}\left(1 + \sqrt{5}\right)$$

$$? \quad \frac{b}{a-b} = \frac{1}{\frac{a}{b} - 1} = \frac{1}{\frac{1}{2}\left(1+\sqrt{5}\right) - 1}$$

$$= \frac{2}{\sqrt{5} - 1} = \frac{2(1+\sqrt{5})}{5-1} = \frac{1}{2}\left(1+\sqrt{5}\right) = \frac{a}{b} = r$$

$$x_1 = a$$

$$x_2 = a + b = a + \frac{a}{r}$$

$$x_3 = a + \frac{a}{r} - \frac{a}{r^2}$$

$$x_4 = a + \frac{a}{r} - \frac{a}{r^2} - \frac{a}{r^3}$$

$$\bar{x} = a\left(1 + \frac{1}{r} - \frac{1}{r^2} - \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} - \frac{1}{r^6} - \frac{1}{r^7} + \cdots\right)$$

$$= a\left(1 - \frac{1}{r^2} + \frac{1}{r^4} - \frac{1}{r^6} + \cdots\right) + a\left(\frac{1}{r} - \frac{1}{r^3} + \frac{1}{r^5} - \cdots\right)$$

$$= a\left(1 - \frac{1}{r^2} + \frac{1}{r^4} - \frac{1}{r^6} + \cdots\right) + \frac{a}{r}\left(1 - \frac{1}{r^2} + \frac{1}{r^4} - \cdots\right)$$

2

$p + px + px^2 + \cdots$

$= \frac{p}{1-x}$

$$= \left(a + \frac{a}{x}\right)\left(1 - \frac{1}{x^2} + \frac{1}{x^4} - \frac{1}{x^6} + \cdots\right)$$

$$= \left(a + \frac{a}{x}\right)\left(\frac{1}{1 + \frac{1}{x^2}}\right) = a\left(\frac{1+x}{x}\right)\frac{x^2}{1+x^2}$$

$$= \frac{ax(1+x)}{1+x^2} = a\,\frac{\sqrt{5}+1}{2} \cdot \frac{1 + \frac{\sqrt{5}+1}{2}}{1 + \left(\frac{\sqrt{5}+1}{2}\right)^2}$$

$1 + \frac{5 + 2\sqrt{5} + 1}{4}$

$\frac{10 + 2\sqrt{5}}{4}$

$$= \frac{a}{2}(\sqrt{5}+1)\,\frac{\frac{3+\sqrt{5}}{2}}{\frac{5+\sqrt{5}}{2}}$$

$$= \frac{a(\sqrt{5}+1)(\sqrt{5}+3)}{2(\sqrt{5}+5)} = \frac{a}{2}\,\frac{(3.23)(5.23)}{7.23} = 1.161\,a$$

1.175

$$x \rightarrow \frac{ax(x+1)}{1+x^2}$$

$$y \rightarrow a - \frac{\frac{a}{2}x(1+x)}{1+x^2} = \frac{a + ax^2 - a - ax}{1+x^2}$$

$$= \frac{ax(x-1)}{1+x^2}$$

$$x' = x - \frac{ax(x+1)}{1+x^2}$$

$$y' = y - \frac{ax(x-1)}{1+x^2}$$

$\rho = ke^{\theta}$, $\log\rho = \log k + \theta$

$\theta = \log\rho - \log k$

$\tan^{-1}\frac{y'}{x'} = \log\rho - \log k$

4

$$x_2' = x_2 - \frac{ar(r+1)}{1+r^2} = a\frac{1+r}{r} - \frac{ar(1+r)}{1+r^2}$$

$$= a(1+r)\left[\frac{1}{r} - \frac{r}{1+r^2}\right] = \frac{a(1+r)}{r(1+r^2)}$$

$$\left\{ \frac{a(1-r)}{1+r^2} \cdot \left(\frac{1+r}{1-r}\ \frac{1}{r}\right) \right.$$

$$y_2' = y_2 - \frac{ar(r-1)}{1+r^2} = a - \frac{a}{r} - \frac{ar(r-1)}{1+r^2}$$

$$= \frac{a}{r}(r-1) - \frac{ar(r-1)}{1+r^2}$$

$$= a(r-1)\left[\frac{1}{r} - \frac{r}{1+r^2}\right] = \frac{a(r-1)}{r(1+r^2)}$$

$$\left. \frac{a(1+r)}{1+r^2} \cdot \frac{(r-1)}{(r+1)r} \right\}$$

$$+\frac{y_2'}{x_2'} = -\frac{\frac{a(1-r)}{r(1+r^2)}}{\frac{a(1+r)}{r(1+r^2)}} = -\frac{1-r}{1+r} = \frac{r-1}{r+1} \quad \left[= \tan\theta = -\cot(\theta - 90°)\right]$$

$$\log\sqrt{x_2'^2 + y_2'^2} = \frac{1}{2}\log\left(\frac{a^2}{r^2(1+r^2)}\left[(1+r)^2 + (1-r)^2\right]\right)$$

$$= \frac{1}{2}\log\frac{a^2(2+2r^2)}{r^2(1+r^2)} = \frac{1}{2}\log\frac{2a^2}{r^2}$$

$$= \log\left(\sqrt{2}\,\frac{a}{r}\right)$$

$$\tan^{-1}\frac{r-1}{r+1} = -\log\sqrt{2}a - \log r - \log R$$

5

Now if $r = \frac{1}{2}(1+\sqrt{5}) \longrightarrow \frac{r+1}{r-1} = \frac{\frac{1}{2}+\frac{1}{2}\sqrt{5}+1}{\frac{1}{2}+\frac{1}{2}\sqrt{5}-1} = \frac{\sqrt{5}+3}{\sqrt{5}-1}$

$= \frac{5+4\sqrt{5}+3}{4}$

$= 2+\sqrt{5}$

and $\frac{1-r}{1+r} = -\frac{r-1}{r+1} = -\frac{\sqrt{5}-1}{\sqrt{5}+3}$

$= -\frac{5-4\sqrt{5}+3}{5-9} = \frac{8-4\sqrt{5}}{4}$

$= 2-\sqrt{5}$

Let $a=1$

? $\tan^{-1}(2+\sqrt{5}) = \frac{\log\sqrt{2}}{\log k}$

$\tan^{-1}(2-\sqrt{5}) = \frac{\log\frac{2\sqrt{2}}{1+\sqrt{5}}}{\log k}$

$\frac{2\sqrt{2}}{1+\sqrt{5}} = \frac{2\sqrt{2}(1-\sqrt{5})}{1-5}$

$= \frac{2\sqrt{2}(\sqrt{5}-1)}{4}$

$= \frac{\sqrt{5}-1}{\sqrt{2}}$

? $\frac{\tan^{-1}(2+\sqrt{5})}{\log\sqrt{2}} = \frac{\tan^{-1}(2-\sqrt{5})}{\log\frac{\sqrt{5}-1}{\sqrt{2}}}$

$\frac{\tan^{-1} 4.236}{\frac{1}{2}\log 2} = \frac{\tan^{-1}(-.236)}{\log(1.236)-\frac{1}{2}\log 2}$

2.236068

69315

$\frac{76°43'}{.34657}$ $\frac{-13°17'}{.21181 \quad -.34657}$

4560
43

.21181

$-.13476$

$\frac{4603}{.3466}$ $\frac{797}{.1348}$

13250 5920

$$\log k = -\tan^{-1}\frac{1+r}{1-r} + \log\sqrt{2} + \log a$$

$$\log k = -\tan^{-1}\frac{r-1}{r+1} + \log\sqrt{2} + \log a - \log\tfrac{1}{2}(1+\sqrt{5})$$

$$\log k = -\tan^{-1}(-2-\sqrt{5}) + \overbrace{\log\sqrt{2} + \log a}^{C}$$

$$\log k - C = \tan^{-1}(2+\sqrt{5})$$

$$\log k - C = -\tan^{-1}(\sqrt{5}-2) - \log\tfrac{1}{2}(1+\sqrt{5})$$

$$= \tan^{-1}(2-\sqrt{5}) - \log(1+\sqrt{5}) + \log 2$$

$$= \tan^{-1}(-.236) - \log(3,236) + \log 2$$

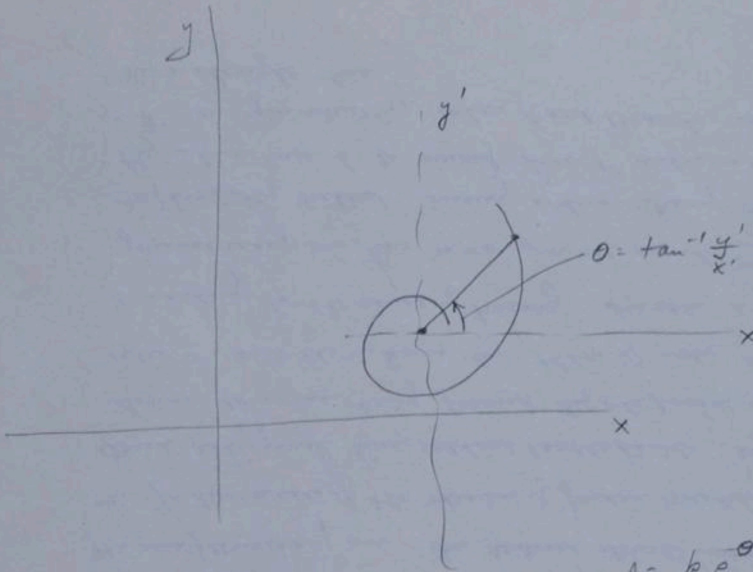
$$= -13.3^\circ -.23 - \quad 1.1725 + .6931 = 1.63$$

$$= \tan^{-1}(4.236)$$

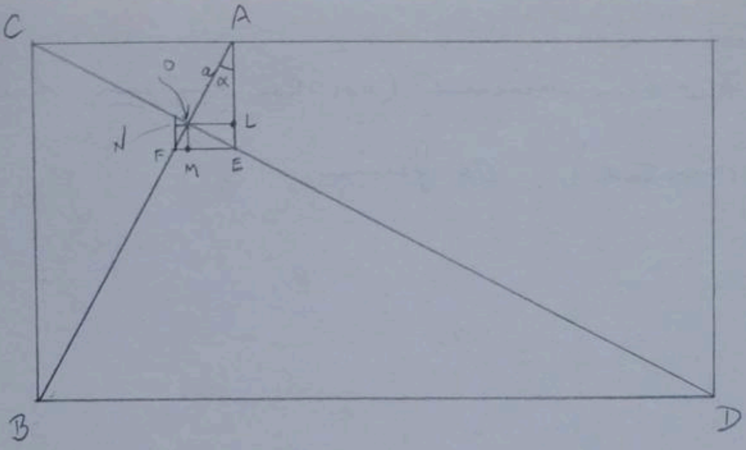
$$= \cancel{77.1} \quad 1.35$$

$$\begin{array}{r} 1.17215 \\ .6932 \\ \hline 1.8657 \\ .23 \\ \hline 1.6357 \end{array}$$

13.3° in rad $= \frac{13.3}{180}\pi$



$\rho = k e^{\theta}$.



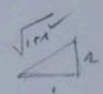
$$\frac{CO}{OB} = \frac{OB}{OD} = r$$

$$\overline{OL}^2 = AL \cdot LE$$

$$\frac{EL}{OL} = \frac{OL}{AL} = r$$

$$OL = r \cdot AL = r \cdot OA \cdot \cos\alpha = a\, r \cos \tan^{-1} \frac{OL}{AL}$$

$$= a r \cos \tan^{-1} r$$

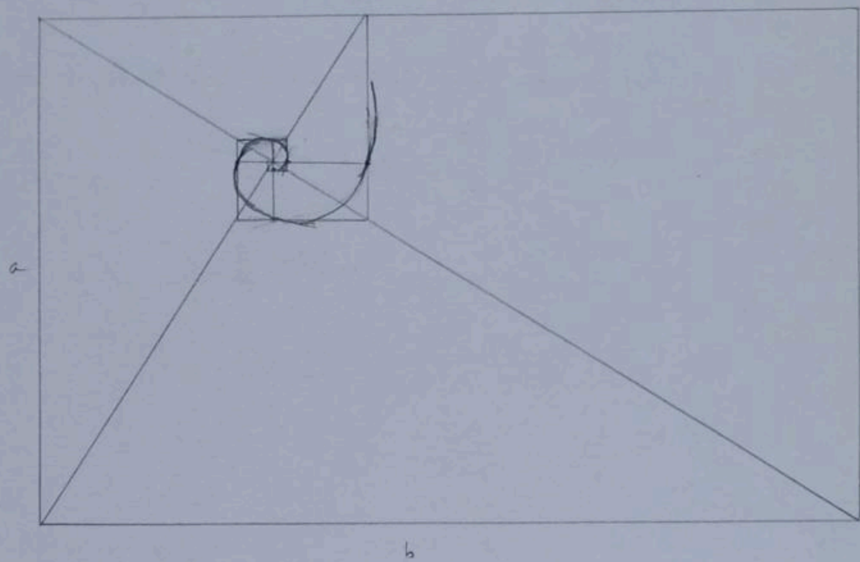


$$= \frac{a r}{\sqrt{1+r^2}} = a \frac{r}{\sqrt{1+r^2}}$$

$$OM = OE \cdot \frac{r}{\sqrt{1+r^2}} = a \tan\alpha \frac{r}{\sqrt{1+r^2}}$$ $$\frac{OE}{a} = \tan\alpha$$

$$= a \cdot r \frac{r}{\sqrt{1+r^2}} = \frac{a r^2}{\sqrt{1+r^2}}$$

nor has any



$\sqrt{5}$ rectangle $\quad \frac{a}{b} = \frac{b}{a+b} = \frac{2}{\sqrt{5}-1} = 1.61\pm$

∴ the pts L, M, N, etc are at distances

$$\frac{ar}{\sqrt{1+r^2}},\ \frac{ar^2}{\sqrt{1+r^2}},\ \frac{ar^3}{\sqrt{1+r^2}},\ \dots$$

from O. Are these pts on a log. spiral

That is can we determine k and μ in

$\rho = k e^{\mu\theta}$ such that

$$\frac{ar^n}{\sqrt{1+r^2}} = k e^{\mu(1-n)\frac{\pi}{2}} \text{ for all } n.$$

If $n=1$ $\quad \frac{ar}{\sqrt{1+r^2}} = k$, hence $\quad r^{n-1} = e^{\mu(1-n)\frac{\pi}{2}}$

For $\quad n=2 \quad r = e^{-\frac{\pi}{2}\mu}$ or $\mu = -\frac{2}{\pi}\log r$.

Will this fit for all n? i.e. is

$$r^{n-1} = e^{\left(-\frac{2}{\pi}\log r\right)\frac{\pi}{2}(1-n)} \quad \text{an} \equiv$$

$$= e^{(n-1)\log r}$$

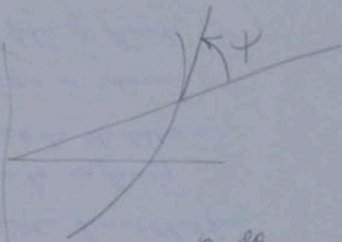
$$= r^{n-1}$$

$$\rho = \frac{ar}{\sqrt{1+r^2}}\, e^{\left(-\frac{2}{\pi}\log r\right)\theta} \qquad \rho = ke^{\mu\theta}$$

$$\tan\psi = \frac{1}{\mu}$$

$$= \frac{ar}{\sqrt{1+r^2}}\, r^{-\frac{2\theta}{\pi}}$$

$$\tan\psi = -\frac{2}{\pi}\log r \qquad \text{if } r \to 1 \quad \psi \to 90° \checkmark$$



Suppose now

$$\frac{CB}{BD} = \frac{BD}{CD+BD} = \frac{\sqrt{5}-1}{2} = r$$

i.e. $r = \frac{\sqrt{5}-1}{2}$

Then $\mu = -\frac{2}{\pi}\log\left(\frac{\sqrt{5}-1}{2}\right) = \frac{2}{\pi}\log\frac{2}{\sqrt{5}-1}$

and $\tan\psi = \frac{\pi}{2\log\frac{2}{\sqrt{5}-1}} = \frac{3.1416}{2\log\frac{2}{1.236}}$

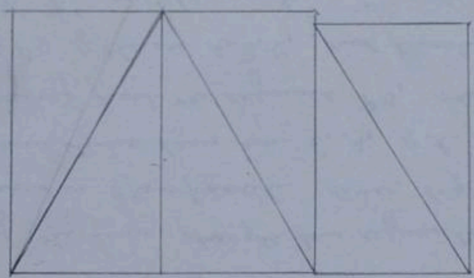
$$= \frac{3.1416}{2(.69315 - .21187)} = \frac{3.1416}{.96257}$$

.21187
.48128

$\psi = 73°$ approx

$$\tan\psi = \rho\frac{d\theta}{d\rho} = \frac{ke^{\mu\theta}}{k\mu e^{\mu\theta}} = \frac{1}{\mu}$$

10.49715 − 10
9.98343 − 10
.51372



Group Connections of n objects

n) $N = 1 + {}_nC_1 + {}_nC_2 + \cdots + {}_nC_n = 2^n - 1$

$n = 1000$ $N >$ 1,000,000,000

If .999 are ineffective there remain 1,000,000 potential

$n = 10$ $N > 1000$

If .99 are effective there remain 10 potential.

important ratio is $\frac{N}{n}$: $1, \frac{3}{2}, \frac{7}{3}, \frac{15}{4}, \frac{31}{5}, \ldots, \frac{1023}{10}$

$$\frac{N}{n} = \frac{2^n - 1}{n}$$

180.65
141.46
322.11
$\boxed{386532}$

40.000
55
2200

$40,000$
4
44000

$(1+a)^n = 1 + {}_nC_1 a + {}_nC_2 a^2 + \cdots + {}_nC_n a^n$

$(1+1)^n = 1 + {}_nC_1 + {}_nC_2 + \cdots + {}_nC_n$

$2^n =$

323.90
322.11
275.40
921.41

1	1	
2	$2(1) + 1(2)$	$= 2^2 - 1 = 3$
3	$3(1) + 3(2) + 1(3)$	$= 2^3 - 1 = 7$
4	$4(1) + 6(2) + 4(3) + 1(4)$	$= 2^4 - 1 = 15$

$n \quad \sum_{i=1}^{n} {}_nC_i - 1 = 2^n - 1$

1	1
2	3
3	7
4	15
5	31
10	$1024 - 1$
100	$(1024)^2 - 1$
1000	$(1024)^3 - 1 \cong 1,000,000,000 - 1$

1000

1,000,000,000
999,000,000
1,000,000

$.999 \times 10^9 =$

999,000,000

228

1.023) 228 (27
2046
7340
7161
1790

88

144

72
160

320
1120

11520

.5 mm

1,75 m = 17500

3500

$(3500)^3 = (3.5)^3 \cdot 10^6$

$= (32)^3 \cdot 10^6$

$= 32.000 \cdot 10^6$

$= 3 \cdot 10^4 \cdot 10^6$

$= 3 \cdot 10^{10} = 30 \cdot 10^9$

214.39
+45.16
168.63
30.00
198.63
203.51

54.12

244.39

3.51

214.39
30
244.39

200.
3.51
203.51
198.63
4.88

[From Folder marked "Copy Ed."] [3 sets of letters]

The Ohio Journal of Science

BUSINESS OFFICE
c/o OHIO ACADEMY OF SCIENCE
505 KING AVENUE
COLUMBUS, OHIO 43201

EDITORIAL OFFICE:
GEOLOGY DEPARTMENT
BOWLING GREEN STATE UNIVERSITY
BOWLING GREEN, OHIO 43402

December 15, 1967

Dr. George B. Van Schaak
Morton Arboretum
Lisle, Illinois

Dear Dr. Van Schaak:

Thank you for your most helpful review of the paper by Dr. Waller. Reviews like this are extremely useful, both to the author and to me, and are, therefore, much appreciated. I also appreciate your kind and sympathetic words about editors. Not everyone feels that way!

A list of all reviewers of papers considered during the year is printed in each January issue of the JOURNAL separate from, and with no reference to, the paper reviewed. If for any reason you do not wish your name included in this list, please let us know.

Sincerely,

Jane L. Forsyth

Jane L. Forsyth
Editor

jcb

3318 Regal Place
St. Louis, Mo. 63139
November 29, 1967

Miss Jane L. Forsyth, Editor
The Ohio Journal of Science
Geology Department
Bowling Green State University
Bowling Green, Ohio 43402

Dear Miss Forsyth,

Thank you for your letter of the 27th regarding Dr. Waller's manuscript. I am naturally pleased to find you sympathic to my earlier report!

I hope the review I have written will be suitable to your uses. I have signed the carbon, thinking that if you needed another clear copy you could xerox the original. I usually like to let things like this 'cook' a while before sending them off--I have a tendency to be too severe in first draft. But this time I cannot wait to think about it, for I might have to rewrite it, and just now I am all out of time.

I resigned as librarian of the Missouri Botanical Garden as of October 1, with some thoughts of retirement. But on September 28 along came the Morton Arboretum, in Lisle, Illinois, asking me to be its first bibliographic consultant. Since then I have been busy selling my house, cleaning up left-overs at the Garden, and packing up my rather considerable library. I expect to get off on the 8th of December, and to start my work on the 18th.

I think it will be an interesting position. There is a large backlog of rarebook cataloging to be done, the necessity to study the whole collection in its relation to what the arboretum should be doing, and as well in its relation to the other botanical collections in the Chicago area.

I hope you detect a certain sympathy on my part with editors, and an interest in editorial work. Should you find it convenient to call upon me again I should be pleased to try to accommodate you.

Sincerely yours,

George B. Van Schaack

The Ohio Journal of Science

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c/o OHIO ACADEMY OF SCIENCE
505 KING AVENUE
COLUMBUS, OHIO 43201

EDITORIAL OFFICE:
GEOLOGY DEPARTMENT
BOWLING GREEN STATE UNIVERSITY
BOWLING GREEN, OHIO 43402

November 27, 1967

Dr. George B. Van Schaack
3318 Regal Place
St. Louis, Missouri 63139

Dear Dr. Van Schaack:

My apologies. I forgot to tell you that the manuscript was sent to us without whatever page was intended to come after page 9. I have not yet been able to obtain whatever the rest is, but I was not concerned because I couldn't believe that whatever was missing could in any way improve the effect produced by what was already available. I feel sure that your review can be completed without that page(s), too.

I am in full sympathy with all your comments about this paper. However, Dr. Waller is a grand old man, so I felt obligated to go ahead and obtain reviews from busy professional people like yourself. I preferred that the refusal not come from me, for after all, I am not a botanist. I appreciate your help in providing me with an evaluation by a professional in this field.

Would you, therefore, prepare a review of this paper, putting down something of what you have said to me, so that I will have something to send to Dr. Waller. As I think you are aware, we encourage reviewers to send us two copies of their evaluation and the unsigned copy is sent to the author, with my editor's covering letter. I will appreciate receiving such a review from you, when you return the manuscript. And let me say again how much I appreciate your cooperation in taking time to read such a paper. I do sympathize, very sincerely, with your reaction to it.

Sincerely,

Jane L. Forsyth

Jane L. Forsyth
Editor

jcb

PUBLISHED JOINTLY BY THE OHIO ACADEMY OF SCIENCE AND THE OHIO STATE UNIVERSITY SINCE 1900

The Ohio Journal of Science

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505 KING AVENUE
COLUMBUS, OHIO 43201

EDITORIAL OFFICE:
GEOLOGY DEPARTMENT
BOWLING GREEN STATE UNIVERSITY
BOWLING GREEN, OHIO 43402

October 20, 1967

Dr. George B. VanSchaack
Missouri Botanic Garden
St. Louis, Missouri 63110

Dear Dr. VanSchaack:

I am enclosing the manuscript that you have agreed to review for THE OHIO JOURNAL OF SCIENCE.

"Dr. George Engelmann and the Saint Louis Road to Modern Botany," by Dr. Aldolph E. Waller.

The Editorial Staff will be very grateful for your opinion regarding the suitability of this paper for publication in the JOURNAL. The author, of course, will appreciate a prompt decision concerning his paper. Four weeks ordinarily is considered an adequate period for completing a review. Most reviewers return the papers sooner.

Please furnish two copies of your specific criticisms or suggestions, one of these to be an unsigned copy, that will be forwarded to the author. The accompanying sheet may be of help in your work of reviewing the paper.

Please return the paper in the enclosed Manila envelope, already stamped and addressed to me.

Thank you very much for your assistance.

Sincerely,

Jane L. Forsyth

Jane L. Forsyth
Editor

jcb

Enclosures 3

3318 Regal Place
St. Louis, Mo. 63139
November 9, 1967

Miss Jane L. Forsyth, Editor
The Ohio Journal of Science
Geology Department
Bowling Green State University
Bowling Green, Ohio 43402

Dear Miss Forsyth,

Your letter and the copy of Dr. Waller's paper came today. Unfortunately the copy seems to be truncated, for there is nothing beyond page nine, which ends in the middle of a sentence. I should be pleased to look at the rest of the paper were you to send it to me.

However, what I have of it enables me to answer questions 1, 2, and 3 of the sheet of 'Suggestions' categorically. This paper is in no sense a contribution to science, being completely derivative from two or three other accounts generally accessible. The author obviously lacks all first-hand knowledge of anything that Engelmann ever did, and fairly obviously has little acquaintance with botany. What he has read is entirely undigested.

But even if this were not so Dr. Waller would have to study the art of writing for a long, long time before he could present a felicitous sentence or a well organized exposition. Compare, for example, the babble beginning on page six, last paragraph, with the well written and pungent paragraphs quoted from Sargent on pages eight and nine—coming upon them is like suddenly hearing beautiful music which til then has been in audible behind a storm of static, abruptly terminated.

I shall be only too glad to document what I mean here by explicit reference to numerous mistakes and to ever present awkwardness, if you wish, but I cannot see how Dr. Waller could possibly rewrite this (far from new) material in any form suitable for a journal of such importance as your own.

Sincerely yours,

George B. Van Schaack

Review of 'Dr. George Engelmann and the Saint Louis Road to Modern Botany' by A. E. Wheeler.

For those aware of the great importance of Dr Engelmann in the history of American science during the 19th century the title of this paper promises a fulfillment of a long cherished wish that his role in botany should be clearly and interestingly presented anew. So ~~[illegible]~~ it is with regret that I must say this paper seems not to have had the benefit of more acquaintance with Dr. Engelmann than can be made ~~found~~ ~~[illegible]~~ in the several somewhat perfunctory accounts already published. Until someone studies the Engelmann reliquary themselves (or part of them) with respect to the botany subjects, which to my fairly certain knowledge no one has ~~yet been~~ done, is anything, properly bearing the paper's title likely to be written. Accordingly I feel this

this highly derivative account can scarcely be considered a 'worthwhile contribution to science'.

Nor do I feel that the presentation of what is offered is either clear or adequate. Both sentence structure and paragraph organization ~~is~~ leave much to be ~~[illegible]~~ ~~statement~~ desired in style, when, even, the facts are not obscured by irrelevant remarks or unbalanced statement. — The first five very brief paragraphs, if such they can be called, have nothing to do with either Dr Engelmann or botany in St. Louis. Despite later ~~the~~ increase in length of paragraphs and more relevance ~~thereafter~~, the general tone of off-hand, unorganized development is [illegible] beginning 'George Engelmann (1809–1884)...' begins with his birth, notes his mother's artistic heritage, makes the unwarranted statement that the University of Heidelberg could not maintain dueling with George Engelmann present, and ends with ~~Engelmann~~ going off to Berlin for two years — all in eleven sentences, three of them averaging nine words each. Such a melange is not the development of any unified aspect of the subject.

Infelicities of expression are numerous: page 1 (3 lines from bottom) 'Equally he was ...' equally with what? p. 2 (end of second paragraph) 'In the only way the University can hope to maintain the establishment'; p. 5 (third paragraph) 'he added more activity'; p. 6 (third paragraph) 'After a few years of his efforts'; p. 7 (third paragraph) 'his merits for naturalism all healed', etc., etc.

I do not see how this paper could be rewritten in suitable form and content without the author making himself much more knowledgeable about Engelmann's actual accomplishments through a personal study of some of them, and without a much ~~greater~~ ~~skill~~ improved skill in presenting material in written form.

~~It is noticed~~ ~~Also~~ It is noticed that the binal part of the paper is lacking, and, here no bibliography is presented for review.

UNIVERSITY OF CALIFORNIA PRESS

BERKELEY • LOS ANGELES • NEW YORK
2223 Fulton Street • Berkeley, California, 94720

November 13, 1967

Mr. George B. Van Schaack
3318 Regal Place
St. Louis, Mo. 63139

Dear Mr. Van Schaack:

Congratulations on what sounds like the opportunity of a lifetime. Your talents will most likely be utilized far closer to capacity than they would be in the rather tenuous connection with us that free-lancing would afford you out here.

I have a hazy recollection that the Arboretum was endowed when I was living in Chicago many, many years ago, and am even more hazy about just how it functions in the botanical world. If the Arboretum issues its own publications I'm sure you could be of great help in getting them out. If not, who knows, maybe you can get some launched as part of your bibliographic contribution. Still, bringing the collection into a coherent whole should be challenging enough.

Please let me extend the very best wishes as you take your new direction. By all means, keep in touch and drop in if you are out here. The door will always be open as long as I'm here.

Yours very truly,

Joel F. Walters

Joel F. Walters

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3318 Regal Place
St. Louis, Mo. 63139
November 9, 1967

Mr. Joel Walters
University of California Press
2223 Fulton Street
Berkeley, California 94720

Dear Mr. Walters,

I've not forgotten your kind offer to let me try my hand at copy editing, and I'd still dearly love to do so—but fate has decided not to let me go to California after all. The afternoon before my last day at the Missouri Botanical Garden with nothing very definite ahead, my telephone rang and the voice on the other end said, 'Say, I just heard you're retiring. We have a library you know, and we want you on our staff'. It was the director of the Morton Arboretum, just west of Chicago. The job was bibliographic consultant, not in the library but directly under him; duties—to catalogue all the beautiful and interesting books Mrs. Zurcher has been buying these many years, to study the whole library collection, and especially what direction the library should take, in particular considering all the other botanical collections in the Chicago area, and to help him formulate new policy for beginning to spend more of the very large income which the institution has. For cream, a house already for me in the arboretum, three hundred yards from the library!

For me it's still a fairy tale, but I've sold my house and have hired the mover for December 6, and I believe I'm going to have a perfectly wonderful time. I'll drop in and tell you all about it the next time I'm fortunate enough to be in Berkeley. In the meantime my regrets that I shan't get to see you sooner.

Sincerely yours,

George B. Van Schaack

3318 Regal Place
St. Louis, Mo. 63139
May 27, 1967

Mr. Philip E. Lilienthal
University of California Press
2223 Fulton Street
Berkeley, California 94720

Dear Mr. Lilienthal,

Thank you for your kind letter of May 19 in which you say you have referred to Mr. Joel Walters my request regarding ~~pos-sible~~ part-time employment. The possibility of acting as a free-lance copy-editor is exactly what I had hoped you might suggest. I look forward to discussing ~~making an appointment~~ with Mr. Walters ~~to dis-cuss~~ your arrangements for this type of work.

I am grateful for your mention of the California Academy of Sciences. I had thought of it from the library standpoint—its important library has been shorthanded as long as I have known it—but it had not occurred to me that the Academy might be looking for some editorial assistance as well. I shall visit there when I am in San Francisco.

Your generous remarks about the pamphlets I had sent you are encouraging.

Sincerely yours,

George B. Van Schaack

UNIVERSITY OF CALIFORNIA PRESS

BERKELEY • LOS ANGELES • NEW YORK

2223 Fulton Street
Berkeley, California • 94720

May 19, 1967

Mr. George B. Van Schaack
3318 Regal Place
St. Louis, Missouri 63139

Dear Mr. Van Schaack:

Thank you for your letter of May 13, outlining your background and experience in editing. We are very glad to have this information and to know you may be settling in this area at a later time.

Unfortunately, we do not anticipate that there will be an opening in our copy-editing department, either full or part-time, in the near future. We do have a rather small volume of work in the natural sciences that we contract for with outside free-lance editors from time to time. This kind of work is extremely intermittent, however, and we could not guarantee that it would offer a livelihood.

Although the above résumé may sound very unpromising, I have referred your letter to Mr. Joel Walters of our copy-editing department. He tells me that he will be on hand during the time you will be attending the ALA conference and will be happy to discuss our copy-editing arrangements in more detail with you if you will call and arrange an appointment. His phone is 845-6000, extension 4252.

In closing, may I ask if you have been in touch with the California Academy of Sciences in San Francisco? It is possible they may be in need of someone with your considerable talents and background. We are returning your booklets under separate cover. It was a privilege to review such well-planned and well-executed publications.

Very truly yours,

Philip E. Lilienthal

PEL:lm

3318 Regal Place
St. Louis, Missouri 63139
May 13, 1967

Mr. Philip E. Lilienthal, Associate Director
University of California Press
Berkeley, California 94720

Dear Sir:

I write you at the suggestion of Mr. Donald Jackson of the University of Illinois Press. I had asked him whether he thought my background, experience, and interests sufficiently broad to induce a press to consider employing me after my near retirement. Our meeting was a chance encounter, and our conversation brief, but he said that I ought to write to you if I were looking for something on the West Coast.

In what my enclosed curriculum vitae says about me I would add just two or three items. Among my acquaintances I am known as something of a perfectionist, and in no respect more so than in the matter of clear exposition. I have written very little myself, but I have read many thousands of words written by others who wanted them critically sifted before they should come into print. I happen also to be one of that melancholy, happily limited, crew whose members cannot avoid seeing the mistake if it is there (and is somebody else's!) I should suppose the most likely type of work a press might find for me would be copy reading or copy editing--a type of work not only adapted to such talents as I may have, but flexible enough to be undertaken at something less than full time.

My normal retirement comes October 1, 1968, but I could retire earlier--in fact, I should like to retire as soon as opportunity for suitable employment might be found. I look for this on the West Coast because of the mild climate and the intellectual environment especially to be found in the Bay area.

I shall be attending the American Library Association Conference in San Francisco, June 25-30, and should appreciate the favor of an appointment with you if my qualifications seem acceptabel as a basis for discussion. Should references be helpful before a meeting I should be glad to supply them.

I enclose several pieces of printing with which I have been concerned and which may be indicative. One of these (Annals of the Missouri Botanical Garden, vol. 50) is in very short supply, and I should appreciate its return in the attached envelope.

Sincerely yours,

George B. Van Schaack

STANFORD UNIVERSITY PRESS

May 16, 1967 Stanford, California 94305

Mr. George B. Van Schaack
3318 Regal Place
St. Louis, Missouri 63139

Dear Mr. Van Schaack:

I have your most ingratiating letter of May 13, and I am sorry to have to reply that we cannot consider you for employment as an editor. If the job were simply a matter of aptitude, or if you were thirty years younger, there would be something to talk about; but the job is a matter of much, much more than aptitude, and there would be no time for you to learn the rest.

I am sorry not to have better news, and wish you the best of luck in your next try.

Sincerely yours,

J. G. Bell

J. G. Bell
Editor

JGB/mhj

P.S. I am returning herewith the material you so kindly sent us.

3318 Regal Place
St. Louis, Missouri 63139
May 13, 1967

Mr. J. G. Bell, Editor
Stanford University Press
Stanford, California 94305

Dear Sir:

I write you at the suggestion of Mr. Donald Jackson of the University of Illinois Press. I had asked him whether he thought my background, experience, and interests sufficiently broad to induce a press to consider employing me after my near retirement. Our meeting was a chance encounter, and our conversation brief, but he said that I ought to write to you if I were looking for something on the West Coast.

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My normal retirement comes October 1, 1968, but I could retire earlier--in fact, I should like to retire as soon as opportunity for suitable employment migh be found. I look for this on the West Coast because of the mild climate and the intellectual environment especially to be fouñd in the Bay area.

I shall be attending the rare book pre-conference meeting of the American Library Association on the Stanford University campus, June 22-24, and should appreciate the favor of an appointment with you if my qualifications seem acceptable as a basis for discussion. Should references be helpful before a meeting I should be glad to supply them.

I enclose several pieces of printing with which I have been concerned and which may be indicative. One of these (Annals of the Missouri Botanical Garden, vol. 50) is in very short supply, and I should appreciate its return in the attached envelope.

Sincerely yours,

George B. Van Schaack

1

[Notes on Cat. Books]
[2 sets]

To catalogue a book means

1. to describe the book in such terms that its identity cannot be in doubt:

2. to record other details respecting it which a possible reader might wish to be aware of <u>before</u> seeing the book:

who wrote it, or who or what was responsible for seeing that it was written; what is its title; when and where did it appear and by whom published; how many pages has it, or into how many volumes is it divided; is it illustrated; what is its size.

source (e.g., editions, detached from a certain journal or reprinted from a certain edition); bibliographies included; important items of contents; etc. (the possible variety is infinite.

3. to provide a classification and associated number so that it can be found on the shelf.

4. to provide added entries (i.e. extra headings on copies of the main card so that people thinking of the book in different ways will find it equally quickly in the catalogue.

The card catalogue and the classification system are the two keys to a library; they each open a door to it ~~the library~~, but in very different ways. Persons unfamiliar with the problems of cataloging (including classification) tend to confuse the descriptive aspects of 1, 2 and 4, and the classificatory aspect of 3.

~~The~~ item 3: ~~concerns~~ two classes of people only: use the information from

a) those who must find the books on the shelves and eventually reshelve them, and

b) those who, having access to the stacks, find their books by consulting the shelves of the class they may think the book ought to be found in. In a personal library this is a fairly ideal system of cataloging, without any use of 1, 2 and 4, for collections under 500, shall we say. Even in ~~[illegible]~~ collections of only a few thousand ~~3000~~ books ~~I find~~ it becomes difficult — there are just too many ways to class a book, ~~even in a fixed system,~~ and ~~even if only two or three differently minded patrons are considered.~~

Then no matter where most books are classed some parts of them are thought of by certain patrons as connected with some other class — few books are so special as to belong to only one clear cut class.

So 1, 2 and 4 play a larger and larger role as the collection grows, and the larger it is the more insistent is the necessity for these items to be well and carefully done.

~~What can is the nature of the operation in 1 and how could a cataloger stand any time on it except to copy the obvious authors name. Choosing~~

Let us look at the first part of item 1, recording on the card who wrote the book. This operation is known as choosing the entry – can ~~these~~ there possibly be any trouble here? No, not when the title page reads: The Palms ~~Volcanoes~~ of the South Pacific by James Stelle. But what about a German book by C. G. F. Schmidt. If we ignore the three given names and use only the initials it seems all must be well. But suppose the C. stands for Carl. Is this man the same as Karl Schmidt for whom already we have catalogued three books. If so why, if not why not? And what about K.W.F. Schmidt. Surely ~~these~~ this man and C.G.F Schmidt must be different people. No, for C. G. F. stands for Carolus Guillelmus Federicus while K. W. F. stands for Karl Wilhelm ~~[illegible]~~ Frederick — they are the same man. If sometimes his book is entered under C. G. F., at others under Karl, at others yet under K. W. F., his various titles will ~~not only~~ be distributed among the various Schmidts; ~~[illegible]~~ no simple alphabetical arrangement of these titles ~~under a uniform entry for him~~ will be possible until a uniform entry for him is established; this is a matter of decision sometimes requiring ~~[illegible]~~ as much as a half ~~extensive~~ {~~[illegible]~~ to a full}, hour of time ~~of tracing your man~~ tracking your man.

The above is not an extreme case. What about a ~~separate~~ Festschrift volume in honor of Carl Schmidt, edited by Wilhelm Trudy, for the Allgemeine Gesellschaft zu Berlin as a special number of its Mitteilungen. No single card can possibly do here: some people remember Schmidt, some Trudy, some the Society, some the city and some an author only who contributed a paper. This book could be entered under Schmidt as the person honored, under Tendy as the person who organized the job, under the Society ~~as a part of the Mitteilungen series~~ which underwrote it; ~~not least~~ – ~~and to take care of all the people who may want to get the book and will remember it in other~~ which shall it be? ~~I am told that~~ Deciding such cases in the Library of Congress, ~~has~~, in certain instances, has required the consultation of three or four cataloguers for a day or more.

And what shall we do about the man who only remembers that Arnold Henry wrote a ~~paper in the~~ paper in ~~the~~ book, and ~~James~~ someone else that there was a paper called 'Tulips and afternoon ...' ~~written on tulips in it~~. If these people are to be served 'added entries' must be provided (i.e. cards like the original with an extra line or two at the top) ~~Dissecond~~ one with ~~Jo~~ Arnold Henry Notes on Cactus at the top, another: Tulips and afternoon leisure at the top, and ad infinitum for all the other people who remember the book in other ways. A line has to drawn somewhere, of course — the ~~tod~~ best lines are drawn by the best cataloguers!!!!

How are we getting along: part way through 1 and conceivably through 4. The rest of 1 isn't so bad, but sometimes it gets pretty messy in detail. 2, as remarked, can be infinite — again, the best cataloguers do the best here.

Finally, to complete 4, what about people who only remember (in the Schmidt case) that lilacs were the subject of some paper. OK, a card with added entry LILAC. NO, no, for we already have a card with SYRINGA; Latin Syringa and English lilacs must be kept together. Do we have a card LILACS? We should,

See
SYRINGA.

and must either check or consult the authority file — a special cataloguer's file which contains all legitimate (i.e. already made in this library) entries of author (etc.), all cross-references of author, all subject headings and their cross-references. ~~etc~~

You cannot give this sort of work to the Melanie Browns of this world, and most definitely not to the Frank Perskés. Good cataloging is a highly skilled endeavour — craft ~~skill~~ or ~~profession, I leave to you to decide~~.

— call it craft or profession as you like — its purpose is to unlock a collection of books for all the people who may want to consult it — a cataloguer's training never ends.

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cop.2
would be such a number. P786 is
the classification number. .A3 denotes
the author's name. N9 denotes the
title's first word. v.1 denotes the
first volume of the set. cop.2 indicates
there are (at least) 2 copies of which
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is to be avoided, with the single exception that the catalogue card may omit both v.1 and cop.2 if the card is to be a collection card referring to the whole set and all copies of it — but, in this case, the card must show that there is a vol. 1 in the set and the shelf list card (a duplicate kept in book number order instead of alphabetical order) must show that there of the several copies of vol. 1 one of them is marked cop. 2.

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4. James Psy
5. Hedda Gabler
6. Tess
7. Leisure Class
8. Man + Superman Shaw
9. Way of All Flesh.
10. Henry James - Golden Bowl.
11. Whitehead Russell
12. Boas - Mind of Prim Man.
13. Henry Adams
14. Psy of Unconscious, Jung
15. Relativity, Sp + Genl Einstein
16. Outline. Wells.
17. Remembrance of Things Past, Proust
18. Babbitt - Lewis
19. ~~Dahn~~ of Dreams Freud
20. Spengler
21. Ulysses
22. Major Int.
23. Rise of Am Civ. Beard
24. Atomic Theory + Desc. of Nature - Bohr.
25. Treat. on Th. of Heat Rad. Planck.

Beard.

1. Marx
2. Bellamy
3. Frazer 1660-1783 Mahan
4. Inf. of Sea Power on Hist
5. Barrack Room Ballads.
6. Leisure Class
7. Imperialism Hobson
8. Jungle - Sinclair
9. Pearson - Elec + Mag.
10. Great Illusion. Angell
11. Married Love + Wise Parenthood Stopes.
12. Imp. Lenin
13. Keynes Econ Cons.
14. Wells Outline
15. Main Street
16. Frontier in Am. Hist Turner
17. Babbitt
18. Pareto
19. Spengler Eddington
20. Int. Const of Stars
21. Now it Can be Told Gibbs.
22. Origin of World War Fay
23. All Quiet
24. Russian Rev. Trotsky
25. My Battle Hitler

Meets.

1. Marx
2. Bellamy
3. Frazer
4. James Psych.
5. Kreutzer Sonata Tolstoy
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17. Keynes.
18. Einstein See over
19. Jean Christophe
20. Main St.
21. Dreams Freud See over
22. Spengler
23. Ulysses.
24. Stars Eddington See over
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[Best Books?]

25 more [in Books

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EDWARD BOK

THE GREATEST AUTHORS, IN POPULAR ESTIMATION, AT LEAST, OF THIS CENTURY

This row, and the one at the bottom of the page, shows the leading figures in a poll undertaken by *The International Book Review*. It is noteworthy that four of the leaders are Englishmen—and that there isn't a woman among them.

WHO ARE THE "CLASSIC AUTHORS" OF TO-DAY?

ONE WAY TO DETERMINE the best books and writers of a generation is by consulting the literary critics. Another way is by putting the matter up to the public at large. Still another is by consulting a more or less picked public, consisting of people, not professional critics, who are fond of reading. This first method, it may be remembered, when applied by *The Literary Digest International Book Review* some time ago, returned Thomas Hardy as the first writer of this generation, and, as the greatest book, his long poetical drama, "The Dynasts." *The Review* has just completed another test, along the third of the three possible lines, by submitting the question to its readers, with results which are summarized in the two tables published on this and page 50 following. To quote from its own account of its investigation:

The International Book Review has just completed a unique poll to determine what its readers consider the best ten books published since 1900. In answer to an editorial invitation extended last July, 1,753 readers of *The International Book Review* have sent in their votes, each naming the ten books of the present century that stand highest in their esteem. The votes for each author and title were carefully tabulated in a card-catalog as they came in, and the final returns show that these 1,753 persons, far from agreeing on any ten, have cast their ballots for no less than 1,201 authors and 2,164 different books.

Certain books, however, have gradually forged ahead in the race, tho some of the favorites have encountered exciting ups and downs. The ten that finally received the highest number of votes—the winners in this wide-spread election—beginning with "The Outline of History" and "The Four Horsemen of the Apocalypse," may be seen in the list displayed in the middle of this page, with the vote cast for each. Representing, as it does, the majority-vote of book-lovers in large centers and remote villages from Maine to California, this list is of unique interest. So far as known, it represents the largest popular literary plebiscite ever held anywhere.

The symposium as planned was based on votes for individual books, and the ten titles on this page are the fruits of that plan. But as the voting proceeded it became apparent that certain authors enjoying the highest popularity were destined not to appear in this list simply because they had written so many good books that their admirers could not agree on any one as the best. This phase of the contest, therefore, is covered by counting the total for each author and making out another list on that basis, as shown on the second page of the present article. Comparing these two lists it will be seen that the title-vote and author-vote are pretty much the same for the first six items, but beyond that point Booth Tarkington, with 342 votes for all his books, overtops O. Henry's 286 votes, and Joseph Conrad's seventeen books bring him a larger vote than all Owen Wister's novels could muster. On this basis John Galsworthy and Sinclair Lewis also may be counted as winners.

There were, of course, thousands of books and authors that

The Ten Books Receiving the Highest Vote

Votes	
563	THE OUTLINE OF HISTORY *By H. G. Wells*
471	FOUR HORSEMEN OF THE APOCALYPSE *By V. Blasco Ibañez*
355	IF WINTER COMES *By A. S. M. Hutchinson*
346	AMERICANIZATION OF EDWARD BOK *By Edward Bok*
345	THE LIFE OF CHRIST *By Giovanni Papini*
302	THE CRISIS *By Winston Churchill*
286	SHORT STORIES *By O. Henry*
281	THE VIRGINIAN *By Owen Wister*
256	LIFE AND LETTERS OF WALTER H. PAGE *By Burton J. Hendrick*
254	THE MIND IN THE MAKING *By James Harvey Robinson*


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BOOTH TARKINGTON

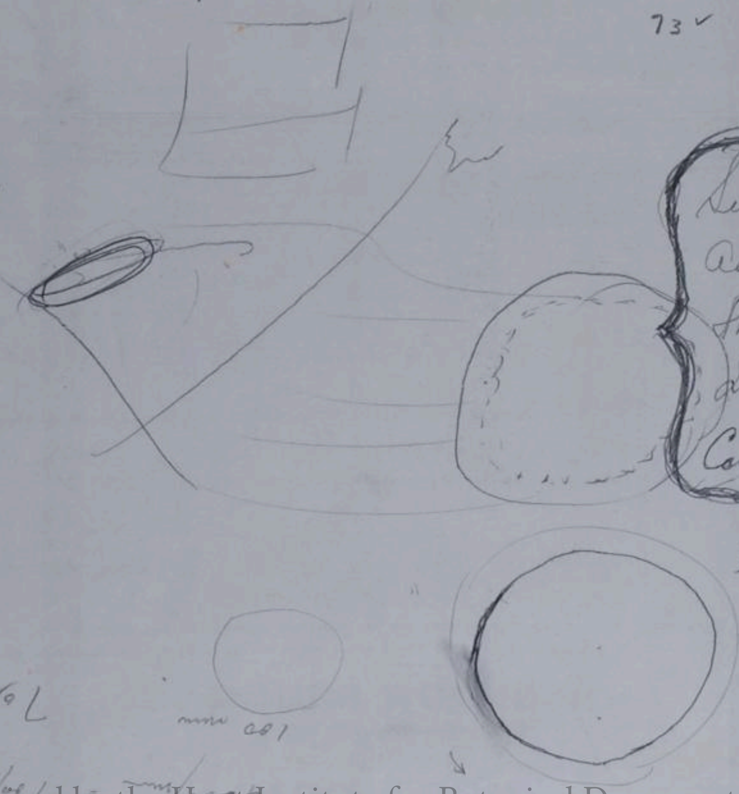

Copyrighted by T. & R. Annan & Sons
JOSEPH CONRAD


Copyrighted by Underwood & Underwood
JOHN GALSWORTHY


SINCLAIR LEWIS

[Bach]

1281	20 ✓
	168 ✓
1301	40 ✓
	70 ✓
1388	109 ✓
	155 ✓
1418	58 ✓
	72 ✓
1484	81 ✓
	187 ✓
1516	205 ✓
1574	34 ✓
	74 ✓
1650	66 ✓
	77 ✓
1671	91 ✓
	122 ✓
1694	178 ✓
	73 ✓



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Bach Cantatas

2 20 3 oboes, VV Va SAB *Very* interesting

7 34 3 trumpets 2 oboes flute timpani Yes
An Easter cantata with a 20 p opening

7 40 Horns oboes VV 20 p chorus, long bass aria
Probably, yes

12 58 with 72 Omit

16 66 With 77 as an alternate

16 70 Yes!

18 72 Omit

18 73 only if with 178

order
20 (168) 1281
34 (74) 1574
40 (70 R) 1301
?
205

alt. 77 (66)
23 (159)
70 (180) only if 1301 not avail

18 74 An Easter Cantata with 34 yes

18 77 with 66 as an alternate

20 81 no

22 91 with 122 no

23 109 no

26 122 no

32 155 no

33 168 with # 20

35 178 no

37 187 no

11 205 YES!

Bach Cantatas (alt.)

70, 180, 23, 159 207a, 214, 210

49, 84, 33, 95, 29, 135

5 23 2 oboes, VVVa, 2 trumb. perhaps as alt.

5 29 w/th 135 3 trumpets 2 oboes, VW " " "

7 33 No No

10 49 No

16 70 As alternate if 1301 not available

20 84 No

22 95 No.

28 135 with 29
159 with 23
35 180 with

207a

~~14~~ 210 where

34? 214

2
5
7
10
11
12
16
18
~~29~~ 20
~~32~~ 22
~~33~~ 23
~~35~~ 26
~~35~~ 28
37

Another fugue on the same subject frequently ascribed to him [see ed. Peters, xv] is definitely spurious. Various later composers have used the famous motive in fugues or fantasias, e.g., Albrechtsberger [cf. *DTOe* 16.ii]; Schumann (*6 Fugen über Bach*, op. 60); Liszt (*Fantasia and Fugue on B.A.C.H.*, for pianoforte and for organ); M. Reger (op. 46); W. Piston (*Chromatic Fantasy on Bach*). It also plays an important part in Busoni's *Fantasia contrappuntistica* (1910).

Bach-Gesellschaft. See *Societies II, 2. Here follows a conspectus of the contents of the edition of the Bach-Gesellschaft, arranged according to subject matter.

I. *Sacred Cantatas.* 1–10: vol. 1.—11–20: vol. 2.—21–30: vol. 5.i.—31–40: vol. 7.—41–50: vol. 10.—51–60: vol. 12.ii.—61–70: vol. 16.—71–80: vol. 18.—81–90: vol. 20.i.—91–100: vol. 22.—101–110: vol. 23.—111–120: vol. 24.—121–130: vol. 26.—131–140: vol. 28.—141–150: vol. 30.—151–160: vol. 32.—161–170: vol. 33.—171–180: vol. 35.—181–190: vol. 37.—191–193: vol. 41.—194: vol. 29.—195–197 (Wedding Cantatas): vol. 13.i.—198 (Trauer Ode): vol. 13.iii.—Unnumbered (doubtful or unfinished): vol. 41. [For the numbering of the cantatas 191ff cf. C. S. Terry, *Bach's Cantata Texts* (1926), 642.]

II. *Secular Cantatas.* 201–205: vol. 11.ii.—206–207: vol. 20.ii (also 34).—208–212: vol. 29.—213: vol. 34.—Unnumbered: vols. 34, 20.ii.

III. *Oratorios.* Weihnachts-Oratorium: 5.ii.—Oster-Oratorium: 21.iii.—Himmelfahrts-Oratorium: 2 (= Cantata 11).

IV. *Passion Music.* Mattheus: 4 (variant of Schluss-choral in 41).—Johannes: 12.i.—Lucas: 45.ii.

V. *Masses and Parts of Masses.* B minor: 6.—F, A, G minor, G: 8.—4 Sanctus: 11.i.—Sanctus, Kyrie, and Christe: 41.

VI. *Magnificat.* 11.i.

VII. *Motets.* Six and two doubtful: 39.

VIII. *Chorales.* 1–185 (Collection of C. P. E. Bach): 39.—3 Choräle zu Trauungen: 13.i. (For a complete collection of Bach's Chorales cf. C. S. Terry, *Bach's Four-Part Chorals.*)

IX. *Songs.* 39.

X. *Orchestral Works* (cl. = clavier; vl. = violin; cont. = continuo). Four Overtures, 1 Sinfonia: 31.i.—Ouverture C moll: 45.i.—6 Brandenburg concertos: 19.—Triple concerto for flute, cl. and vl.: 17.—7 Concertos for one cl.: 17.—3 Concertos for 2 cl.: 21.ii.—2 Concertos for 3 cl.: 31.iii.—Concerto for 4 cl.: 43.i.—2 Concertos for vl.: 21.i (also 45.i).—Sinfonia for vl.: 21.i.—Concerto for 2 vl.: 21.i.

XI. *Chamber Music.* Sonata for flute, vl. and cont.: 9.—Trio, Canon for flute, vl. and cont. (from the Musical Offering): 31.ii.—Instrumentalsatz für vl., Hoboe und Cont.: 29.—7 Sonatas for cl. and vl. (one doubtful): 9.—Suite for cl. and vl.: 9.—Sonata, Fugue for vl. and cont.: 43.i.—Sonata for two vl. and cont.: 9.—4 Inventions for vl. and cl.: 45.i.—Sonata in G for vl. and cont.: Neue B.-G. 30, I.f. i.—3 Sonatas, 3 Partitas for vl. solo: 27.i.—6 Suites for cello solo: 27.i.—3 Sonatas for gamba and cl.: 9.—3 Sonatas for cl. and flute: 9.—3 Sonatas for flute and cont.: 43.i.

XII. *Clavier Music.* Six English Suites: 13.ii (new ed. in 45.i).—6 French Suites: 13.ii (new ed. in 45.i; fragments in 36).—6 Partitas: 3.—French Overture: 3.—Miscellaneous suites (fragments): 36, 42, 45.i.—Overture: 36.—Inventions, 2- and 3-part: 3.—Well-tempered Clavier: 14 (Variants: 36; Autograph: 45.i).—7 Toccatas: 3 and 36.—Sonatas: 36, 42, 45.i.—Italian Concerto: 3.—16 Concertos (Vivaldi): 42.—Goldberg Variations: 3.—Aria variata: 36.—2 Capriccios: 36.—Chromatic Fantasia: 36.—Preludes (Fantasia) and fugues, Preludes, Fantasias, Fugues: 36.—4 Duets: 3.—Clavier Uebung i,ii,iii: 3.—Notenbuch der Anna Magdalena Bach (1722, '25): 43.ii.—Clavierbüchlein für W. F. Bach (1720): 45.i.

XIII. *Organ Music.* Seventy Chorale preludes (46 Orgelbüchlein; 18 Choräle; 6 Schübler): 25.ii.—21 Chorale preludes (from Clavierübung iii): 3.—65 Chorale

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preludes (24 Kirnberger; 28 others; 13 doubtful): 40. — [10 Chorale preludes not included in B.-G. are reprinted in Ed. Peters, vol. 9]. — 6 Sonatas: 15. — 4 Concertos (after Vivaldi): 38 (variant in 42). — [2 other Concertos in Ed. Peters, 9]. — 18 Preludes and fugues: 15. — 3 Preludes and fugues: 38. — 3 Toccatas: 15. — Passacaglia: 15. — 8 Kleine Praeludien und Fugen: 38. — 6 Fantasias, 3 Preludes, 6 Fugues, Canzona, Allabreve, Pastorale, 2 Trios: 38. — [2 other Trios in Ed. Peters, 9].

XIV. *Art of Fugue.* 25.i (original form: 47). — Musical Offering: 31.ii, — Canons: 45.i.

Bach trumpet. See under *Clarin trumpet.

Backfall. English 17th-century name for the *Appoggiatura. See also *Appoggiatura, Double II.

Badinage, badinerie [F., playfulness, banter]. A dance-like piece of jocose character which occurs as a movement in the optional group of the 18th-century suites, e.g., in Bach's Suite in B minor.

Bagatelle [F., a trifle]. A short piece, usually for the pianoforte. The name was used by François Couperin ("Les Bagatelles," see his *Pièces de Clavecin*, new ed. Augener, ii, ordre 10) and, in particular, by Beethoven, whose *Bagatellen* (op. 33, op. 119, op. 126) mark the beginning of the extensive literature of 19th-century *character pieces.

Bagpipe [F. *musette*; G. *Dudelsack, Sackpfeife*; It. *piva, zampogna*]. Generic name for a number of instruments which have one or (usually) several reed-pipes attached to a windbag from which the air is blown into the pipes; also, specifically, the name for the Irish and Scottish varieties of this family. [See the illustration on p. 152 (Clarinets).] One or two of the pipes, called chanter (chaunter), are provided with soundholes and are used for melodies, while the other, larger ones, called drones, produce one tone each and are used for the accompaniment. In the earlier, Eastern specimens, both chanter and drones are clarinets (i.e., have single reeds) while in the modern types either they are both oboes (i.e., with double reeds), as in Italy and some parts of France, or the drones are clarinets while the chanter is an oboe, as in Scotland, Ireland, Brittany. Two categories of bagpipes may be distinguished, according to whether the wind in the bag is provided from the mouth through an additional blowing-pipe, or by a small pair of bellows placed under and operated by the arm. To the former type belong the *Old Irish bagpipe*, the *Highland bagpipe* (Scotland), the *biniou* (Bretagne), the *cornemuse* (France), the *Dudelsack* or *Sackpfeife* (Germany), the *zampogna* and *piva* (Italy); to the latter: the *Northumbrian bagpipe* (England), the modern *Irish bagpipe*, the *gaita* (Galicia), the **musette* (France). A more primitive instrument was the *bladder pipe*, a single or double clarinet with a bladder used as a bag [illustrated in *GD*, pl. LX].

The bagpipe was not known to the Babylonians, Jews, and Greeks, but was used in Rome (*tibia utricularis*). Nero is reported to have played on it. In the Middle Ages it is frequently mentioned under different names (*musa, chorus, symphonia, chevrette*). The famous illuminations of the 13th-century Spanish MS *Escorial j b 2* [see *Cantiga] show players of bagpipes [cf. *GD* iv, 184; *ReMMA*, 222]. In the British Isles the bagpipes have played, for many centuries, a prominent role in folk music and in military music. Their continental history is less interesting, except for a late 17th-century movement in France which, for a short time, raised the instrument to a standing in society and in art music [see *Musette]. See also *Pibroch.

Lit.: Wm. H. Grattan-Flood, *The Story of the Bagpipe* (1907); W. L. Manson, *The Highland Bagpipe* (1901); G. Askew, *A Bibliography of the Bagpipe* (1932).

Baguette [F., stick]. Drumstick (— *de bois*, wooden drumstick; — *d'éponge*, sponge-headed drumstick). Also the ba-

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S. NIMSGERN, *Bass* - Frankfurt Kantorei
Bach Collegium Stuttgart - H. RILLING, *Cond.*

MHS 1281

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No. 70: "Wachet! Betet! Betet! Wachet!"

M. FRIESENHAUSEN, *Sop.* - H. LAURICH, *Alto* - H. KUNZ, *Bass*
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MHS 1301

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No. 159: "Schet, wir gehn hinauf gen Jerusalem"

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J. STAEMPFLI, *Bass* - Frankfurter Kantorei - Deutsche Bach-Solisten
K. THOMAS, *Cond.*

MHS 1242

No. 207 a: Auf, schmetternde Töne der muntern Trompeten
No. 214: Tönet, ihr Pauken! Erschallet, Trompeten

I. REICHELT, *Sop.* - E. LISKEN, *Alto* - G. JELDEN, *Ten.*
E. WOLLITZ, *Bass* - Kantorei Barmen-Gemarke
H. KAHLHOFER, *Cond.*

MHS 1375

"Unschuld! Kleinod reiner Seelen" - ARIA for Soprano, 2 Flutes & Oboe da caccia
No. 210 (Wedding Cantata): "O holder Tag, erwünschte Zeit"

U. BUCKEL, *Sop.* - Deutsche Bach-Solisten
H. WINSCHERMANN, *Cond.*

MHS 1474

No. 49: "Ich geh' und suche mit Verlangen"
No. 84: "Ich bin vergnügt mit meinem Glucke"

A. GIEBEL, *Sop.* - J. STÄMPFLI, *Bass* - Westphalian Kantorei
W. EHMANN, *Cond.*

MHS 1386

No. 109: "Ich glaube, Lieber Herr"

H. LAURICH, *Alto* - K. EQUILUZ, *Ten.*

No. 155: "Mein Gott, wie lang', ach lange"

I. REICHELT, *Sop.* - N. LEHRER, *Alto* - F. MELZER, *Ten.*
H. KUNZ, *Bass* - Gächinger Kantorei - Bach Collegium, Stuttgart
H. RILLING, *Cond.*

MHS 1388

No. 33: "Allein zu dir, Herr Jesu Christ"
No. 95: "Christus, der ist mein Leben"

G. BERNAT-KLEIN, *Sop.* - E. BORNEMANN, *Alto*
G. JELDEN, *Ten.* - R. KUNZ, *Bass* - Bremer Cathedral Choir
Bremer Bach Orchestra - H. HEINTZE, *Cond.*

MHS 1400

No. 29: "Wir danken Dir, Gott, wir Danken Dir"
No. 135: "Ach Herr, mich armen sunder"

J. WEHRUNG, *Sop.* - J. HOEFFLIN, *Ten.* - E. LISKEN, *Alto*
J. STÄMPFLI, *Bass* - South German Madrigal Choir
Deutsche Bach-Solisten - W. GONNENWEIN, *Cond.*

MHS 1417

An alphabet of whit, being ~~[illegible]~~ scantily annotated selections of especially useful words found in a miniature dictionary for travellers to Holland, together with examples for their ~~usage~~ likely employment. Not to be confused with Voltaire's An alphabet of wit; more humorous, more interesting and more useful. [Please note carefully the force of the semicolon!]

atrabilious. zwartgallig. For the more erudite speaker.

His atrabilious temperament reigning, he saw no cause for laughter.

bream. brasem. For ichthyophiles or ichthyophages.

The vessel was brimful of bream.

chilblained hands. winter handen. For winter travellers only.

With chilblained hands he tightened the straps on his skates.

~~Selections~~ Selections from a ~~miniature~~ dictionary for travellers to Holland, ~~together with~~ examples of their usages. (An alphabet of whit. *)

maculation — bevlekking

~~Particularly useless in Holland where~~

~~An~~ Especially useful ~~want~~ used & in low grade restaurants where the tablecloth is apt to be soiled, hence particularly useless in Holland. The maculation of the pause was a painful reminder of the ~~former~~ recent presence of ~~____~~ mucosa.

Alow — beneden. True, Holland is a low country, but avoid using beneden in this sense.

atrabilious — zwartgallig. ~~Only the most~~ For the more erudite speaker.

~~and otherwise melancholy spirits~~

~~need pause here.~~

His atrabilious temperament ~~____~~ reigning, he ~~he~~ saw no cause for laughter.

chilblained hands — winterhanden. For winter travellers only. With chilblained hands he ~~____~~ tightened ~~the~~ the ~~____~~ straps ~~____~~ on his skates.

dotation — begiftiging. Do not confuse with vergiftigen, to poison. The dotation ~~____~~ implicit in connection will only later be realized.

bream — brasem. For ~~ichthyological experts~~ ichthyophiles, (or ichthyophobes)

~~The dish~~ The vessel was brimful of bream.

* Being whitfully annotated selections of especially useful words found in a miniature

Not to be confused with Voltaire's An alphabet of wit; more humorous, more interesting and more useful [~~___~~ Please note carefully the force of the semicolon!]

jemmy. breekijzer. Not a toothpick!

Use the <u>jemmy</u>, Jemmy, and jemmy it.

knight errant. dolende ridder. s.a. [see Eng/Neth.]

The <u>knight errant</u> good for his cough.

~~linger dealer~~

lactiferous. melkgevend. For dairymen and midwives and others.

Chicory is a <u>lactiferous</u> plant.

monastery. klooster. s.o. [see knight errant]

The man was ~~[illegible]~~ very <u>monastery</u>.

~~sweat~~ ~~perspiration~~

~~zebra~~ ~~zebra~~
zephyr gachtwindje

yore oudetijd ~~not a pronoun~~
used as possessive pronoun in
Haarlem only ~~[Eng.~~ [~~not~~ in Haarlem]

x none Nooit te verfoneelwise [Haarlem.

walking gentleman figurant masculine for
walking lady.
shouted 'The walking gentleman, 'Cheese it, the cops!' and fled in terror.'
virago hellevaeg [from hell-a-vague]
A hell-of-a-vague word'?
you don't know what it
means.

unclose openen Here the Dutch is more direct.

transude doorzweeten recondite form of ooze

sagemilk saliemelk Exact signification undetermined;
not in Webster or Century; cf.
lactiferous.

rapeseed raapzaad ~~Watson~~ ~~[illegible]~~?
aphrodisiac?

quarter day betaaldag A point in time and not a
six hour span.

pair of nut-crakers ~~notenkraker~~ see nutcrackers, pair of

otherwhere eldera Some otherwhere otherwhere
is elders.
If otherwhere you roam
there is no place like home'

nut-crakers, pair of see pair of nut-crakers

Missouri Botanical Garden
2315 Tower Grove
St. Louis, 10, Mo., USA
February 24, 1964

Wheldon + Wesley, Ltd
Lytton Lodge, Codicote
Hitchin, Herts England

Dear Sirs:

Please send on my account one copy of E. S. D. Warne, A Russian Scientific Reader, Allen + Unwin (Spring Books 1964) 10 s. net

Sincerely yours

George B. Van Schaack
GEORGE B. VAN SCHAACK

expectorate. slijm opgeven. Polite form for spit.

To expectorate is not polite.

fripper. uitdrager. No comment:

Having lost his shirt he ~~made a visit~~ recovered
~~to the~~ it at the fripper's.

dotation. begiftiging. Do not confuse with vergiftigen, to poison.

The dotation implicit in conception will only later be realized.

expectorate. slijm opgeven. Polite form for 'spit'.

To expectorate is not polite.

frippery. uitdrager. s. a. [i.e., sine annotatione]

Having lost his shirt he recovered it at the fripperie.